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SOME ASPECTS OF THE NETWORK ANALYSIS OF SEQUENCE TRANSDUCERS

BY

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SUMMARY

An algebraic formulation is presented for use in the analysis of networks of the synchronous type of sequence transducer discussed by Mealy (1).[2] With the network analysis problem in mind, the operation of a sequence transducer is characterized by two matrices and the formulation is given for elementary cases of cascade, parallel, and feedback connections. This formulation involves matrix-type operations and derives the characterizing matrices for the over-all network from those of the component sequence transducers. The formulations are intended to serve as a framework for the general network analysis problem. The analysis problem for feedback connections is also discussed.

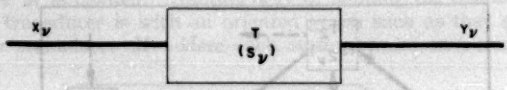

FIG. 1. Diagram of a sequence transducer.

INTRODUCTION

Consider the two-pole T in Fig. 1 wherein the input is a synchronous type of sequence with values ranging over a discrete alphabet. The value of the input during the νth interval of time will be designated by x_ν. The operation of T is regarded as being described by the relations introduced by Mealy (1) for the synchronous type of sequence transducer which, in the present notation, are:

$$y_\nu = f(x_\nu, s_\nu) \tag{1}$$

$$s_{\nu+1} = g(x_\nu, s_\nu). \tag{2}$$

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

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In these relations, y_ν and s_ν are the values during the νth interval of time of the output and state of T, respectively, and they also range over discrete alphabets. The alphabets of the input, output, and state of T are of lengths a, b, and c, respectively. There is an understood order for these alphabets so that to establish this order it will be convenient to use for the alphabets the numerals $0,1,2, \ldots, (L-1)$; $L = a$, b, or c.

In general, the alphabet lengths a, b, and c may not be finite and the point functions f and g may vary with ν or may even be nondeterministic (random) functions, where f and g are characterized by the conditional probability distributions of y_ν and $s_{\nu+1}$, respectively, given the values of x_ν and s_ν. However, in the present paper, a, b, and c will all be regarded as being finite and the functions f and g as being time-invariant and deterministic.

There are different possibilities for the physical nature of the quantities in Fig. 1. The input may or may not change from one value to another in negligible time. It may always have a "rest" value between symbols. Also, the time durations of the input, output, and state symbols may or may not precisely coincide. These questions must be considered in detail in physically realizing a sequence transducer, but are not dealt with here, ideal adherence to the operation defined by Eqs. 1 and 2 being assumed. It is of interest to note that any sequence transducer whose operation can be defined by Eqs. 1 and 2 is physically realizable by the method of Mealy (1).

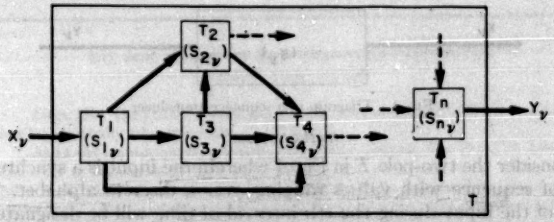

FIG. 2.—General network of sequence transducers.

The general problem of network analysis can be described as follows. Consider the network T in Fig. 2 consisting of an interconnection of two or more sequence transducers. The objective is to derive the relations which describe the over-all operation of T through a simple algebraic formulation. The component transducers in Fig. 2 are seen in general to have multiple inputs, but by interpreting all combinations of inputs as different members of a single alphabet the component transducers can then be considered to have a single input as does the

simple transducer depicted in Fig. 1 and defined by Eqs. 1 and 2. An important step in the problem is the selection of an appropriate algebraic formulation for the operation of the single sequence transducer, and this is considered first.

REPRESENTATION OF OPERATION

The functions f and g in Eqs. 1 and 2 that characterize a sequence transducer may be expressed in several ways, the most direct being simply a tabulation of the functions for each occurring combination of present input and state. This may be done by listing all the combinations in successive rows and giving the values of present output and new

S_y	X_y	Y_y	S_{y+1}
0	0	0	1
0	1	1	1
1	0	1	0
1	1	0	1

FIG. 3. Tabulation defining M.

state in each row. An example is given in Fig. 3 wherein sequence transducer M is defined. Another way of defining the operation of a sequence transducer is with an oriented graph such as that shown in Fig. 4 for transducer M. Here each state appears as a node. The

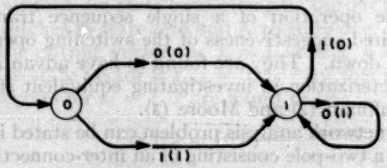

FIG. 4. Oriented graph for M.

notation on any path depicts the input-output relationship with the input involved being in parenthesis, while the node to which the path leads depicts the new state and the path from which the path emanates the "present" state.

Vector and matrix forms of the tabulation in Fig. 3 are introduced to meet the requirements for the algebraic formulations desired here. These are similar to those introduced by Huffman (2) for the asyn-

chronous type of sequence transducer. The output matrix is defined as the matrix whose entry in the ith row and jth column is the value of the present output as depicted by Eq. 1 for the i and j values of the present state and input, respectively. The state matrix is defined as the matrix whose entry in the ith row and jth column is the new state as depicted by Eq. 2 for the i and j values of the present state and input, respectively. The output and state matrices will collectively be called the operation matrices. The operation matrices for transducer M defined in Fig. 3 are shown in Fig. 5, in which (M_ν) and $(m_{\nu+1})$ are,

$$(M_\nu) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad (m_{\nu+1}) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

FIG. 5. Operation matrices for sequence transducer M.

respectively, the output and state matrices. Each row in the state matrix will be called the state vector for the corresponding state, each row in the output matrix will be called the output vector for the corresponding state, and the output and state vectors for a given state will collectively be called the operation vectors for that state. To illustrate, the output and state vectors for state one of transducer M defined in Fig. 3 are, respectively:

$$(M_\nu)_1 = (1\ 0) \tag{3}$$

$$(m_{\nu+1})_1 = (0\ 1). \tag{4}$$

The operation vectors and matrices are introduced above for the network analysis problem, but they are also found to be quite useful for defining the operation of a single sequence transducer. They provide the desired suggestiveness of the switching operations and are very readily set down. They are found to have advantages over other means of characterization in investigating equivalent states, a subject discussed by Huffman (2) and Moore (3).

The general network analysis problem can be stated in the following terms. Given: a two-pole consisting of an inter-connection of sequence transducers. Derive: the operation matrices of the two-pole from the operation matrices of the component transducers. This paper develops several techniques for the solution of some simple forms of this problem and suggests a method of approach for the case where there are feedback connections.

CASCADED CONNECTIONS

Let sequence transducers M and N be cascaded as in Fig. 6, the νth state of M being m_ν and the νth state of N being n_ν. Let (M_ν) and

$(m_{\nu+1})$, respectively, be the output and state matrices of M, and (N_ν) and $(n_{\nu+1})$, respectively, be those of N. The objective is to determine output and state matrices, designated respectively as (z_ν) and $(s_{\nu+1})$, defining the transducer NM characterized by the total state $s_\nu = m_\nu, n_\nu$. The alphabet of the output of M must evidently be contained in the alphabet of the input of N.

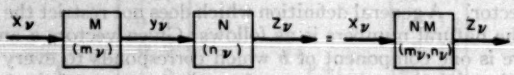

FIG. 6. Cascade connection of sequence transducers.

For simplicity, the discussion will be supplemented with an illustration where transducer M is that previously defined by the operation matrices in Fig. 5 and transducer N is defined by the operation matrices in Fig. 7. It is understood that the inputs to M to which the com-

$$(N_\nu) = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad (n_{\nu+1}) = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}$$

FIG. 7. Operation matrices for sequence transducer N.

ponents of its operation matrices refer are successive values of x and the inputs to N are the successive values of y. A direct procedure for deriving the desired operation vectors for the over-all network is not difficult to perceive. For example, consider the case where $s_\nu = 1, 2$. If the input zero is applied the output of M (in state one) is one, and this becomes the input to N (in state two) giving an output of zero. At the same time, M switches to state zero and N to state one. Thus, the first component of the output vector for $s_\nu = 1, 2$ is $z_\nu = 0$, and the first component of the state vector is $s_{\nu+1} = 0, 1$. By this process the output and state vectors for $s_\nu = 1, 2$ are found to be:

$$(z_\nu)_{1,2} = (0\ 0) \tag{5}$$

$$(s_{\nu+1})_{1,2} = (0,1\ 1,0). \tag{6}$$

There is economy in defining vector operations which give algebraic formulation to the cascading process, making it more methodical, and this will now be considered. An operation will first be defined which will be called the ordering operation and will be denoted by the symbol $\odot$. It is by definition an operation between two vectors: for example, a and b, and it yields a vector; for example, c, the operation being depicted by $c = a \odot b$. If $a = (a_0\ a_1)$ and $b = (b_0\ b_1)$, then $c = (c_0\ c_1)$

where $c_0 = b_{a_0}$ and $c_1 = b_{a_1}$; that is, c_0 is the a_0th component of vector b, etc. To illustrate, let $a = (1\ 0)$ and $b = (2\ 3)$. Then,

$$c = a \odot b = (1\ 0) \odot (2\ 3) = (3\ 2). \tag{7}$$

The ordering operation is seen to yield a vector having the same number of components as the first vector but having entries taken from the second vector. A general definition which does not restrict the alphabet of a to the natural numbers is as follows: Given vectors a and b such that there is one component of b which corresponds to every value of the alphabet for the components of a, then $c = a \odot b$ is defined by selecting for the ith component of c the value for that component of b which corresponds to the value of the ith component of a, with i taking on as many values as there are components in a. The ordering operation provides the essential mechanism for obtaining the output vector which may be expressed for state i, j of the cascaded transducers as:

$$(z_r)_{i,j} = (M_r)_i \odot (N_r)_j. \tag{8}$$

That is, the output vector of cascaded transducers in a given over-all state $s_r = i,j$ is determined by the ordering operation between the output vector of M for the state i and the output vector of N for the state j. To obtain $(z_r)_{1,2}$ in the above example by the ordering operation formula, Eq. 8 yields:

$$(z_r)_{1,2} = (M_r)_1 \odot (N_r)_2 = (1\ 0) \odot (0\ 0) = (0\ 0) \tag{9}$$

which agrees with the result in Eq. 5. This process can be repeated to obtain all $(z_r)_{i,j}$, $i = 0,1$ and $j = 0,1,2$. To derive the over-all state matrix, a second operation called the adjacency operation will be defined between two vectors of the same length and denoted by a semicolon between the two vectors. When the operation is between vectors a and b, it yields a vector c, the operation being depicted by $c = a\,;\,b$. If $a = (a_0\ a_1)$ and $b = (b_0\ b_1)$ then the adjacency operation between a and b yields:

$$c = (a_0\ a_1)\,;(b_0\ b_1) = (a_0,b_0\ a_1,b_1). \tag{10}$$

That is, the first component of c is the ordered pair consisting of the first component of a and the same component of b, etc. The over-all state vectors can now be obtained from the vector equation:

$$(s_{r+1})_{i,j} = (m_{r+1})\,;((M_r)_i \odot (n_{r+1})_j). \tag{11}$$

Thus, for the state $s_r = 1,2$ in the above example,

$$\begin{aligned}(s_{r+1})_{1,2} &= (0\ 1)\,;((1\ 0) \odot (0\ 1)) = (0\ 1)\,;(1\ 0)\\ &= (0,1\ 1,0)\end{aligned} \tag{12}$$

which agrees with the result in Eq. 6. From the formulas in Eqs. 8 and 11, the total operation matrices can be directly derived and the result is shown in Fig. 8 in which the order of the states is understood to be 0,0, 0,1, 0,2, 1,0, 1,1, 1,2.

$$(Z_\nu) = \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad (S_{\nu+1}) = \begin{pmatrix} 1,0 & 1,1 \\ 1,1 & 1,2 \\ 1,0 & 1,1 \\ 0,1 & 1,0 \\ 0,2 & 1,1 \\ 0,1 & 1,0 \end{pmatrix}$$

FIG. 8. Operation matrices for cascade network.

It may be noted that if the highest numbered entry of $(M_r)_i$ ever exceeded the number of components in either $(N_r)_j$ or $(n_{r+1})_j$ for any i and j, then an undefined condition would result in the above vector formulas. That this cannot happen can be seen to be guaranteed by the requirement that the length of the input alphabet of N equals or exceeds the length of the output alphabet of M.

The solution of the converse problem, that of decomposing a given transducer into two cascaded transducers, which presumably would be simpler ones, would be very useful in both the problems of synthesis and analysis. The sequence transducer, however, does not appear suited to straightforward decomposition operations.

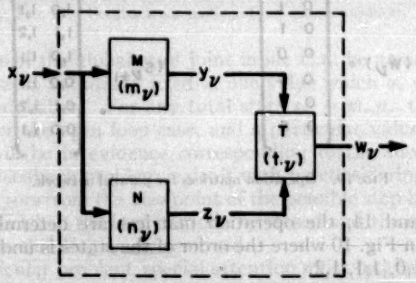

FIG. 9. Parallel connection of sequence transducers.

PARALLEL CONNECTIONS

Consider Fig. 9 in which sequence transducers M and N, in states m_r and n_r, respectively, are supplied with a common input. Their combined outputs form the input to sequence transducer T. The

objective is to derive the operation matrices for the total sequence transducer (shown dashed in Fig. 9) in terms of the total internal state depicted by m_ν, n_ν, t_ν. The output vectors $(M_\nu)_i$, $(N_\nu)_j$, and $(T_\nu)_k$, and the state vectors $(m_{\nu+1})_i$, $(n_{\nu+1})_j$, and $(t_{\nu+1})_k$ are given for all i,j,k. The output vector for the over-all network in the state i,j,k will be depicted by $(w_\nu)_{i,j,k}$ and the state vector by $(s_{\nu+1})_{i,j,k}$.

The inputs to T are values of y_ν, z_ν and the alphabet of this joint input is understood to be ordered with y_ν and z_ν progressing through their respective values in some standard fashion. Thus, if the alphabets of both y_ν and z_ν are 0,1, then the alphabet of the joint quantity y_ν, z_ν can be considered to be ordered 0,0, 0,1, 1,0, 1,1. Consider where the over-all network is in its i,j,k state. Derive the vector $(M_\nu)_i;(N_\nu)_j$, the operation being the adjacency operation. Then the ordered pairs in this vector represent the present (joint) input to transducer T. It is then apparent that:

$$(w_\nu)_{i,j,k} = ((M_\nu)_i;(N_\nu)_j) \odot (T_\nu)_k \tag{13}$$

$$(s_{\nu+1})_{i,j,k} = (m_{\nu+1})_i;(n_{\nu+1})_j;\{[(M_\nu)_i;(N_\nu)_j] \odot (t_{\nu+1})_k\}. \tag{14}$$

As an example, let transducers M and N be those defined above and, for simplicity, let transducer T be the one-state sequence transducer ("combinational network") defined by the output vector $(T_\nu) = (0\ 0\ 0\ 1)$, the state matrix in this case being trivial. The state of the over-all network is designated by simply $s_\nu = m_\nu, n_\nu$. Then, applying Eqs. 13 and 14, the operation matrices are determined and the result shown in Fig. 10 where the order of the states is understood to be 0,0, 0,1, 0,2, 1,0, 1,1, 1,2.

$$(W_\nu) = \begin{bmatrix} 0 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (S_{\nu+1}) = \begin{bmatrix} 1,0 & 1,1 \\ 1,1 & 1,2 \\ 1,0 & 1,1 \\ 0,0 & 1,1 \\ 0,1 & 1,2 \\ 0,0 & 1,1 \end{bmatrix}$$

FIG. 10. Operation matrices for parallel network.

FEEDBACK CONNECTIONS

In the problems considered above the inter-connections did not include feedback paths. A final consideration in this paper will be for the elementary inter-connection that does contain such paths as illustrated in Fig. 11. The analysis of feedback networks for sequence

transducers, including the determination of stability and the derivation of the over-all network operation matrices, when a stable solution exists, generally presents a difficult problem with practical circuits. A brief outline of a method of analysis is presented here for the sample network in Fig. 11. The solution is obtained by first determining the matrices of the simpler open loop connection resulting from breaking the connection at point P in Fig. 11 and allowing external inputs u_ν to be applied along with inputs x_ν. The feedback quantity u_ν' is observed along with the transducers' state transitions. From these open loop characteristics, it is then possible to consider the step by step switching that results when the loop is closed, facilitating the solution of the closed loop problem.

Consider the open loop network described above. The determination of feedback quantity u_ν' and the corresponding new state $s_{\nu+1}' = m_{\nu+1}, n_{\nu+1}$ as functions of the total input x_ν, u_ν and of the present state $s_\nu' = m_\nu, n_\nu$, is the problem in cascaded transducers. The solution of this simple cascading problem yields the operation vectors for each state.

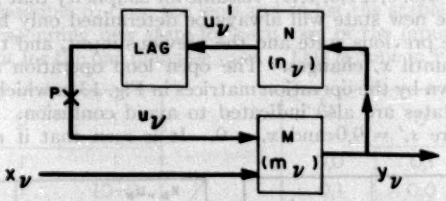

FIG. 11. Feedback connection of sequence transducer.

Let the order of the alphabet of joint input x_ν, u_ν be arranged for these vectors so that x_ν is initially fixed at one value which u_ν varies through its separate alphabet. For any total state $s_\nu' = m_\nu, n_\nu$, the prime here being used for the open loop case, and a particular value of input x_ν a sub-vector will be in evidence corresponding to the successive values of u_ν. The feedback problem is dealt with by considering every one of these sub-vectors from the viewpoint of the possible step by step switching operations when the loop is suddenly closed and the external source for input u_ν is simultaneously removed.

In a particular problem, special attention must be given to the exact physical nature of the transducer in studying the step by step switching operations described. The lag noted in Fig. 11 was assumed in the above discussion so that the input to the transducer M in Fig. 11 could be regarded as being fixed long enough for the cascaded transducers M and N to operate in a predictable manner. If at any time in Fig. 11 u_ν' does not equal u_ν, then if the loop is closed at P, u_ν will be changed

to u_ν' after only the delay caused by the lag network. The network is considered to be in stable operation when after only a reasonably small number of transitions a value of u_ν' occurs equal to the value of u_ν which contributed to it. The duration of the input x_ν is regarded as sufficiently long for such switching to be completed.

To illustrate the above procedure, let transducers M and N be defined as in Fig. 12 in which the order of the inputs to M is understood

$$(M_\nu) = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix} \qquad (N_\nu) = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

$$(m_\nu + 1) = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix} \qquad (n_\nu + 1) \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

FIG. 12. Operation matrices for sequence transducers M and N.

to be $x_\nu, u_\nu = 0,0, 0,1, 1,0, 1,1$. Assume for simplicity that M and N are such that the new state will always be determined only by the stable values of the previous state and the previous input, and the state will not change until x_ν changes. The open loop operation as described above is shown by the operation matrices in Fig. 13 in which the present input and states are also indicated to avoid confusion. Consider in Fig. 13 where $s_\nu' = 0,0$ and $x_\nu = 0$. It is seen that if $u_\nu = 0$, then

x_ν, u_ν / s_ν'	$x_\nu = 0$		$x_\nu = 1$	
	$u_\nu = 0$	$u_\nu = 1$	$u_\nu = 0$	$u_\nu = 1$
0,0	1	1	1	1
0,1	1	1	1	0
1,0	1	1	1	1
1,1	0	1	0	0

(u_ν')

x_ν, u_ν / s_ν'	$x_\nu = 0$		$x_\nu = 1$	
	$u_\nu = 0$	$u_\nu = 1$	$u_\nu = 0$	$u_\nu = 1$
0,0	1,1	0,1	0,1	1,1
0,1	1,0	0,0	0,0	1,1
1,0	0,1	1,1	1,1	0,1
1,1	0,1	1,0	1,1	0,1

$(s_\nu' + 1)$

FIG. 13. Open loop operation of feedback network.

$u_\nu' = 1$, while if $u_\nu = 1$, $u_\nu' = 1$. In the latter case, if the loop were closed, a stable condition occurs, whereas, in the former case, a temporarily unstable condition occurs in which, after a lag, u_ν changes from zero to unity where a stable condition occurs. Thus, for $s_\nu' = 0,0$ and $x_\nu = 0$, stable closed loop operation results in which the feedback u_ν always arrives at the value $u_\nu = 1$. It is apparent that for any combination of the input x_ν and the state s_ν', if u_ν' equals u_ν for one value

and not for the other, a single stable condition will result for closed loop operation. Examples of such cases in addition to the above are where $x_\nu = 0$ and $s_\nu' = 0,1$ and 1,0 and where $x_\nu = 1$ and $s_\nu' = 0,0$ and 1,0 and 1,1.

Exceptional cases occur in Fig. 13 first for $x_\nu = 1$ and $s_\nu' = 0,1$ where u_ν' never equals u_ν, and second for $x_\nu = 0$ and $s_\nu' = 1,1$ where u_ν' always equals u_ν. In the former case, if $u_\nu = 0$, $u_\nu' = 1$ and after a lag u_ν will change to unity after which u_ν' will change to zero and u_ν will soon change back to zero, etc. This oscillation will occur for the entire duration of x_ν. The new total state would be 0,0 or 1,1 depending upon which case of u_ν is binding, and in this case the future operation will be considered as indeterminate when the combination of $x_\nu = 1$ and $s_\nu' = 0,1$ occurs. The other case, in which u_ν equals u_ν' for both values, evidently means that if the previous value of u_ν was unity, it will remain at unity, and if zero will remain at zero. These two stable conditions for the same value of x_ν are uniquely different situations and one must be represented by an added possible state for the over-all connection as will be considered.

It is now possible to close the loop and to list the stable value of u_ν' and the corresponding new state for each case of the input x_ν and the state s_ν' except for the one unstable case. This is done in Fig. 14 where

s_ν \ x_ν	0	1
0,0	1	1
0,1	1	(0-1)
1,0	1	1
1,1′	1	0
1,1″	0	0

(u_ν)

s_ν \ x_ν	0	1
0,0	0,1	1,1′
0,1	0,0	(0,0-1,1′)
1,0	1,1′	0,1
1,1′	1,0	1,1″
1,1″	0,1	1,1″

$(s_{\nu+1})$

FIG. 14. Closed loop feedback quantity and transitions.

the derivation of entries from Fig. 13 are self-evident except for the phenomenon of new states 1,1′ and 1,1″ and the entries for $s_\nu = 0,1$ and $x_\nu = 1$. The latter case is the oscillating case and this is depicted by indicating the entry which gives the two possibilities with parenthesis. States 1,1′ and 1,1″ were added as a consequence of there being two different stable values of u_ν' as was seen in Fig. 13 for its state 1,1 with input zero. State 1,1 is seen to be caused under conditions with $u_{\nu-1}$ at values of both zero and one. When $u_{\nu-1} = 1$ the present system state is called $s_\nu = 1,1'$ and, when $u_{\nu-1} = 0$, the system state is called $s_\nu = 1,1''$. Thus, whenever a transition from a stable condition to state 1,1 is indicated in the matrix $(s_{\nu+1}')$ in Fig. 13, then the state

transition in the matrix (s_{y+1}) in Fig. 14 is either 1,1′ or 1,1″ accordingly as the stable output u_y' corresponding to the occurrence of state 1,1 in Fig. 13 is one or zero. State 1,1′ will therefore only occur with u_y initially at one and state 1,1″ with u_y initially at zero, so they are each stable and unambiguous.

$$(y_y) = \begin{pmatrix} 0 & 1 \\ 0 & (0-1) \\ 0 & 1 \\ 0 & 1 \\ 1 & 1 \end{pmatrix} \qquad (s_{y+1}) = \begin{pmatrix} 0,1 & 1,1' \\ 0,0 & (0,0-1,1') \\ 1,1 & 0,1 \\ 1,0 & 1,1'' \\ 0,1 & 1,1'' \end{pmatrix}$$

FIG. 15. Operation matrices for feedback network.

The final result, namely the output matrix for the true output of the feedback network, (y_y), is next obtained quite simply from Figs. 12 and 14 and is given in Fig. 15. Here, the order of the states is understood to be 0,0, 0,1, 1,0, 1,1′, 1,1″.

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PROPERTIES OF THE EXPONENTIAL DISTRIBUTION OF EXPONENTIAL POPULATIONS

BY

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ABSTRACT

Properties of the negative exponential distribution of exponential populations are calculated as an illustration of the behavior of a population in which the average extension of life for the survivors increases as a function of the time. The equation specifying the distribution is

$$dN/dt = -N_0 \int_0^\infty ke^{-kl-1/l}dl/l$$

and the n^{th} moment about the origin is given by

$$\mu' = (n!)^2/k^n.$$

This population has a greater proportion of individuals of short life than a corresponding simple exponential; at intermediate times there are fewer observations, and for a time of ten or twenty average lives the number of survivors is orders of magnitude larger than for the simple exponential distribution. The properties of the population surviving at time T are computed using the K_n arising from Bessel's functions of an imaginary argument. A striking property is that the survivors at time T have an expectation of life which is given by $T + (1/k) + T^{1/2}K_0(4kT)^{1/2}/k^{1/2}K_1(4kT)^{1/2}$, which is greater by $(T/k)^{1/2}K_0(4kT)^{1/2}/K_1(4kT)^{1/2}$ than the result obtained for the exponential. A distribution obtained in experimental work, which appears to be adequately represented by the postulated distribution, is compared with it.

PART I—DEVELOPMENT OF STATISTICS FOR THE POPULATION AND FOR THE SURVIVORS

Introduction

In any simple exponential distribution each individual has a constant probability of transforming during each second until it disappears from the population. The individuals surviving at any time have the same probability of transforming as each individual in the population had originally. As measured from this time, their average added life which we call the expected extension of life, is the same as that of the entire population originally. The expectation of life for these individuals, therefore, is just the time that they have survived plus the average life in the exponential population from which they come.

Although the expected extension of life for the individuals at any time in a simple exponential population is independent of the time, there are numerous illustrations in nature where the expected extension of life increases with the time. Whenever a population consists of two or more exponential distributions with different decay constants the expected extension of life increases with the time. This occurs because

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of the rapid elimination from the population of those individuals having a short average life and the resulting increase in the proportion of individuals of long average life. In the theory of phosphorescence, for example, electron traps of different depths may be postulated (1).[2] Exponential decay is expected for the electrons in each set of traps separately. At a given temperature, however, the deeper traps have a much lower decay constant than the shallow traps. The total decay may thus be the sum of several simple exponentials. There is some evidence (2) that the exponential distribution of traps is not uncommon, and Burgess (3) has applied the statistics of the exponential distribution of exponential populations to analysis of the properties of semicon ductors.

Some years ago one of the writers studied the decay of electrons trapped on the surface of a nickel grid (4). This decay changed so gradually over wide ranges that it could be interpreted as due to traps having a continuous distribution in depth. More recently in this laboratory we have engaged in statistical studies, under various conditions, of the distribution of times which a low pressure mercury arc persists after it has been established. In general with fixed ambient temperature and circuit conditions, the lives are exponentially distributed; so that if the logarithm of the number of tests in which the arc survives to a given time is plotted as a function of this time, a linear relationship is shown. The decay constant varies rapidly with the arc current and there is a marked dependence on whether or not the spot is anchored. We have obtained results in which the distribution of arc lives cannot be represented by a simple exponential. In some cases the expectation of life has increased unmistakably with a delay in the operation of the timing mechanism sufficient to eliminate numerous short lives. Not infrequently there have been circumstances such that after an arc had persisted to the average life, it was more likely than not to persist an additional time equal to the average life. The most reasonable interpretation of such experience is that we have inadvertently obtained conditions in which the decay constants in various tests were themselves distributed statistically.

It is our purpose in this paper to analyze the properties of a distribution in which individual species would show an exponential distribution, but in which species of short average life are relatively more abundant according to a simple negative exponential law. We assume that the decay of any individual removes it permanently from the distribution we are considering. Furthermore, for calculating the basic functions we shall assume that we deal with such extremely large numbers of individuals that statistical fluctuations are entirely negligible, and our decay law applying at all times is definite and exact. The expected statistical fluctuations in a finite sample are easily incorporated when we have need for them.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

Moments of the Exponential Distribution of Exponential Populations

Let l represent the average life of a given species with individual observed lives distributed exponentially. Then the decay of this species is given as a function of time by

$$dN = -F(l)e^{-t/l}dt/l,$$

where $F(l)$ is the number of individuals at $t = 0$ having average life l. [The number of individuals of this species decaying in a long time, as determined by integration over t from 0 to ∞, is $F(l)$.] In general $F(l)$ can be chosen arbitrarily. It can, for example, be zero everywhere except at one definite value of l where it becomes equal to the number of individuals initially in the population. This choice gives rise to the simple exponential population. $F(l)$ can, however, be some continuous function of l. For the purposes of this analysis we assume that it is a simple negative exponential function of l such that the number of individuals in the range dl at l is N_0kdle^{-kl}, where N_0 represents the total number of individuals of all species initially in the population. Thus for the individuals of all species

$$dN/dt = -\int_0^\infty N_0ke^{-kl-t/l}dl/l. \tag{1}$$

The fact that the constants have been chosen to normalize the distribution is easily verified by the integration

$$N = N_0\int_0^\infty kdle^{-kl}\int_0^\infty e^{-t/l}dt/l,$$

which gives $N = N_0$. This means that in infinite time all of the individuals existing at $t = 0$ transform. If Eq. 1 is divided by N_0, the proportion of the population in dl at l and in dt at t is obtained.

The various moments of this distribution about the origin are basic to the analysis of the properties of the population. The n^{th} moment μ_n' is given by

$$\begin{aligned}\mu_n' &= \int_0^\infty kl^ndle^{-kl}\int_0^\infty e^{-t/l}\left(\frac{t}{l}\right)^n\frac{dt}{l} \\ &= [n!]^2/k^n.\end{aligned} \tag{2}$$

This result, of course, compares with $n!/k^n$ for the simple exponential. It is seen that the higher moments for the exponential distribution of exponential populations increase much more rapidly than in the case of the simple exponential. The first moment about the origin gives the average life or the expected life for an individual selected at random

from the entire population. This is $\mu_1' \equiv l = 1/k$, and it agrees exactly with that for the simple exponential, a result which is not accidental since the distribution function was chosen so that the average life would be $1/k$ just as for the simple exponential. The second moment, however, is $4/k^2$ as compared with $2/k^2$ for the simple exponential.

Having obtained the moments about the origin, and incidentally the mean of the distribution, we can determine the moments μ_n about the mean from the general principles of mathematical statistics.

$$\mu_n = \mu_n' - \frac{n}{k}\mu_{n-1}' + \frac{n(n-1)}{2!k^2}\mu_{n-2}' - \cdots$$

$$= \frac{[n!]^2}{k^n} - \frac{n[(n-1)!]^2}{k^n} + \frac{n(n-1)}{2k^n}[(n-2)!]^2 - \cdots. \quad (3)$$

The Distribution of Survivors and its Various Moments

This distribution presents some very interesting problems, the solutions of which involve integration over finite ranges. For a first example: what is the number or proportion of individuals surviving to a definite time? For a second example, an individual of the population picked at random is observed for the time T, during which it does not transform. What is the expected extension of life for this individual? If T is set equal to zero, the previous analysis applies. If T is finite, however, the chance that the individual comes from a species of short life is reduced and the expected extension of life is increased above the value $1/k$, which of course, would be obtained for any individual picked at any time from the survivors of a simple exponential population.

Let p represent the proportion of the individuals surviving until the time T. Then

$$p = \int_0^\infty kdle^{-kl}\int_T^\infty e^{-t/l}dt/l$$

$$= \int_0^\infty kdle^{-(kl+T/l)}. \quad (4)$$

The behavior of the integrand is interesting. On account of the factor $\exp(-T/l)$ it starts with a zero value. It is this decrease in the proportion of the survivors having short average lives that gives rise to the very great increase in the expectation of life for the survivors. The value of l at which the integrand attains a maximum value can be obtained by setting the derivative of the integrand equal to zero. Thus we obtain $l_m = (T/k)^{1/2}$, and at this value of l, the integrand has the value $k\exp(-2\sqrt{kT})$. At values of l very much smaller than $(T/k)^{1/2}$, the integrand varies essentially as $\exp(-T/l)$. At values much larger than $(T/k)^{1/2}$, the integrand varies essentially as $\exp(-kl)$.

Fortunately it is possible to evaluate the integral of Eq. 4 in terms of tabulated functions. The form of the integrand suggests the use of Bessel's functions of an imaginary argument. Let

$$\theta \equiv \int_0^\infty e^{-(kl+T/l)}dl.$$

Then

$$\int_k^\infty \theta dk = \int_0^\infty e^{-(kl+T/l)}dl/l.$$

Let

$$l = \sqrt{\frac{T}{k}}\,e^t \quad \text{and} \quad dl = e^t dt\sqrt{T/k},$$

so that

$$\begin{aligned}\int_k^\infty \theta dk &= \int_{-\infty}^\infty e^{-\sqrt{kT}(e^t+e^{-t})}dt \\ &= \int_{-\infty}^\infty e^{-2\sqrt{kT}\,\cosh\,t}dt \\ &= 2\int_0^\infty e^{-2\sqrt{kT}\,\cosh\,t}dt. \end{aligned} \tag{5}$$

Now the functions K_n are given by (5)

$$K_n(x) = \int_0^\infty e^{-x\,\cosh\,\alpha}\cosh\,(n\alpha)d\alpha. \tag{6}$$

Thus

$$\int_k^\infty \theta dk = 2K_0(2\sqrt{kT}).$$

Differentiation with respect to k and multiplication by k gives

$$p = 2\sqrt{kT}\,K_1(2\sqrt{kT}). \tag{7}$$

Even though in the details of our analysis, small differences between relatively large quantities arise, tables (6) give K_n to more places than we require for $x = 0.1$ to $x = 10.0$ in increments of 0.1 in x. The problem of determining the proportion of the original population surviving to the time T and other related statistics of the distribution is therefore in essence solved.

The number of survivors in a sample of 1000 observations (neglecting the effect of statistical fluctuations) is indicated by the smooth curve drawn in Fig. 1. Here $3 + \log_{10} p$ is plotted on the y-axis as a function of T. Also in this same figure a straight line indicates the behavior of a simple exponential distribution of the same average life. It is seen

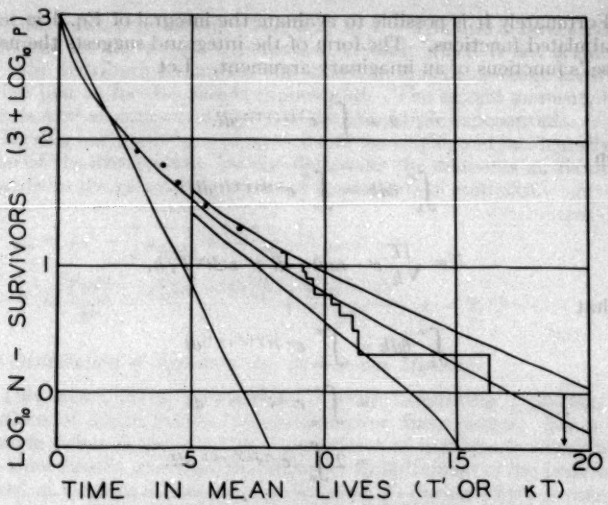

FIG. 1. Number of survivors as a function of time in the exponential distribution of exponential populations.

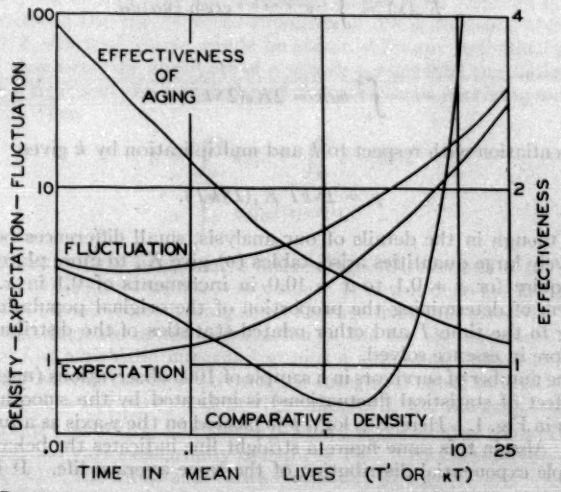

FIG. 2. Statistics of the exponential distribution of exponential populations.

that the exponential distribution of exponential populations gives numerous short observations. For this effect there is compensation in the fact that at times of several average lives, the number of survivors is distinctly larger than in the case of the simple exponential population, and that at times as great as ten average lives, the proportion surviving is orders of magnitude greater than for the simple exponential. It is to be noted that of the number surviving the proportion transforming in the time dT at T is given by the slope of the curve shown in Fig. 1. Differentiation of p gives

$$\frac{dp}{dT} = -\,2kK_0(2\sqrt{kT}). \tag{8}$$

The ratio $(dp/dT)/-\,k\exp(-\,kT)$ called "comparative density" of observations is plotted on the logarithmic scale on the left-hand side of Fig. 2 as a function of time also represented on a logarithmic scale.

The expectation of life for the survivors and the statistical fluctuation in the lives of the survivors are important properties which can be obtained from the moments about the origin. The techniques of calculation are those illustrated in the above development. For the first moment of the survivors at the time T about the origin we obtain

$$\mu_1' = 2\left\{\left[kT^{3/2} + \left(\frac{T}{k}\right)^{1/2}\right]K_1(2\sqrt{kT}) + TK_0(2\sqrt{kT})\right\}. \tag{9}$$

The expectation of life for the survivors is

$$\begin{aligned} l(k,T) &= \mu_1'/p \\ &= T + \frac{1}{k} + \left(\frac{T}{k}\right)^{1/2} K_0(2\sqrt{kT})/K_1(2\sqrt{kT}). \end{aligned} \tag{10}$$

This quantity is plotted on a logarithmic scale shown on the left-hand side of Fig. 2 against the time T as abscissas, also plotted on a logarithmic scale to facilitate presentation of data over a wide range of variation.

The statistical fluctuation in the lives of the individuals depends upon the second moment of the survivors. About the origin this moment can be expressed in a variety of ways, and in terms of K_0 and K_1 we obtain

$$\begin{aligned} \mu_2' = &\left[8\frac{T}{k} + 4T^2\right]K_0(2\sqrt{kT}) \\ &+ [6T^{5/2}k^{1/2} + 8T^{3/2}k^{-1/2} + 8T^{1/2}k^{-3/2}]K_1(2\sqrt{kT}). \end{aligned} \tag{11}$$

By a basic elementary theorem of moments, we obtain the second moment about the mean life of the survivors. Then $\overline{(l - \bar{l})^2} = \mu_2/p$. The standard deviation in the lives of the survivors is $\sigma = (\mu_2/p)^{1/2}$. It is μ_2/p, called the fluctuation, which is plotted in Fig. 2.

Further Properties of the Distribution of Survivors

Aside from p, the proportion of the population which survives to time T and the expectation of life for these survivors, (Eqs. 7 and 10), perhaps the most interesting property is the dependence on T of the quantity $d\bar{l}/dT$. It is found that

$$\frac{d\bar{l}(T,k)}{dT} = \frac{K_0(2\sqrt{kT})}{K_1(2\sqrt{kT})}\left[\left(\frac{1}{kT}\right)^{1/2} + \frac{K_0(2\sqrt{kT})}{K_1(2\sqrt{kT})}\right]. \qquad (12)$$

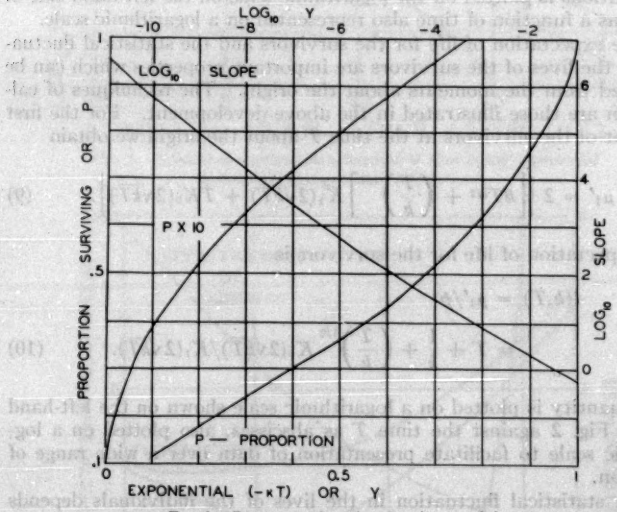

FIG. 3. Representation of proportion surviving.

Calculations from this equation indicate that the properties of the exponential distribution of exponential populations are such that aging the population by the time ΔT always adds more than ΔT to the expectation of life for the individuals surviving at the end of the interval ΔT. The quantity shown in Eq. 12, which we call "effectiveness of aging," also has been plotted as a function of T in Fig. 2. When T is small $d\bar{l}$

can be several times dT. For example, taking $1/k$ as our unit of time ($T' \equiv kT$) and specifying that $T' = 0.01$, we find that dl/dT' is 3.8, while at $T' = 25$, dl/dT' falls to 1.1.

An interesting representation of the data results from the transformation $y = \exp(-kT)$. The range of T from zero to infinity corresponds to a variation of y from unity to zero. Thus the behavior of p as a function of T over the infinite range is compressed into the range of y from 0 to 1 as shown on the axis of abscissas in Fig. 3. The scale of ordinates is shown on the left-hand side of the figure. An effort has been made to increase the accuracy of the representation for small values of y by plotting the second curve marked "$p \times 10$" on a scale for which *both* coordinates are multiplied by 10 before plotting the quantity. The chief limitation of this mode of representation is that the slope at the origin (not shown) is so large that p would appear to rise vertically as a function of y. In Fig. 3 also $\log_{10}$ of dp/dy, called the slope, is plotted on the scale shown at the right-hand side of the figure against $\log_{10} y$ as abscissas shown at the top of the figure. It is seen that for a y of about 0.01, the slope dp/dy is about 1, and for y equal to about 10^{-10} the slope dp/dy is about 10^8.

PART II—EXPERIMENTAL WORK AND COMPARISON OF PROPERTIES SHOWN BY THE DISTRIBUTIONS

An experimental tube, for the purpose of obtaining the distribution of arc lives, was prepared by sealing tungsten electrodes as shown in

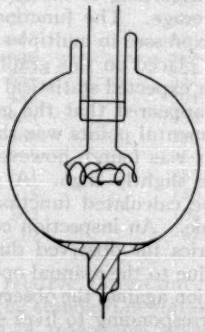

FIG. 4. Experimental arc tube.

Fig. 4 into a pyrex glass bulb. A piece of misch metal was placed in the tungsten filament. The tube was placed on a vacuum line consisting of a mercury vapor diffusion pump backed by a rotary mechanical pump. After evacuation of the bulb, and before mercury was added by dis-

tillation, the tungsten filament was heated to a temperature sufficient to vaporize some of the misch metal. This resulted in a film which was very largely cerium metal being deposited on the inside wall of the experimental tube. Mercury was then added by distillation until a suitable cathode pool was formed. An electric arc was set up within the tube and it was operated with an unusually large current for about an hour while still on the pumps. It was then sealed from the vacuum line and placed in an electric furnace. The furnace was raised to a fixed temperature for the making of observations. In order to secure durations of such length that they could readily be timed with a stop watch, the resistance in series with the arc was set at 2000 ohms and the current was set at 250 ma. To start the arc, a variac supplying the plate transformer of the rectifier-filter system providing the direct current for the tube was turned up until at a potential of about a thousand volts, a glow was established. The transition to the arc under existing circuit conditions was almost immediate, and when the cathode spot had been established, the current was higher than that desired. The variac was then turned down until 250 ma was indicated on a meter in series with the experimental tube. As soon as the desired current was obtained, the timing was started and it was not stopped until the arc went out. The process was repeated until one thousand observations, sufficient to establish the nature of the distribution with some certainty, were recorded.

The arithmetical mean of all observations was obtained. Recorded lives were then classified according to the time of survival in intervals simply related to the average. The function $\log_{10} 1000\ p$ was then plotted against the time expressed in multiples of the average life. The experimental points were placed on this graph as suggested by Fig. 1, and making allowances for expected statistical fluctuation in the sample of 1000 observations, it appeared that the general shape of the plot resulting from the experimental points was the same as that resulting from the calculations. It was found, however, that at low times the experimental points were slightly high. A reason for this was not difficult to discover. The calculated function and its negative slope are both high at zero time. An inspection of the experimental data, however, showed no entries for observed durations less than half a second. It is clear that due to the manual operation of the equipment there will be a discrimination against the observation of very short lives, and that observations corresponding to lives shorter than the reaction time of the observer will be eliminated from the set. The nature of the simple exponential distribution is such that the loss of a fraction of a second from an observation due to a delay in starting the timing is immaterial, because the expected extension of life is the same as long as the arc is on whenever the timing is started. This condition does not apply here. It seems reasonable to assume that the effective size of our

sample should be increased by the number of observations that according to calculation should have fallen between zero time and 0.25 sec. A calculation based on Eq. 7 shows that this number is 22 observations. The calculated curve therefore should be increased by 2.2 per cent throughout its entire range. Although this is not shown in Fig. 1, the difference is carried in the calculations and tabulations.

The result of statistical fluctuation should be incorporated into our calculations. Each point on the calculated curve for p gives the probability of observations longer than the designated time. When multiplied by N_0 it gives $\bar{a}_r$, the average number of individuals falling into this classification in a large number of tests. The statistical fluctuation is given by

$$(a_r - \bar{a}_r)^2 = \bar{a}_r(1 - \bar{a}_r/N_0).$$

The square root of this is the standard deviation σ. As an indication of the uncertainties due to statistical fluctuations, the standard deviation is added and subtracted from the result calculated on the basis of the infinite sample.

TABLE I.—*Comparison of Observed Number of Survivors with Calculated Number.*

	Survivors		
T	Observed	Calculated	σ
0.0889	808	801	12
0.1270	749	750	13.5
0.1799	677	685	14.5
0.2116	642	653	15
0.2540	608	612	16
0.3598	534	532	16
0.4232	485	504	16
0.5100	433	450	16
0.6348	388	396	15.5
0.7217	361	366	15
1.020	261	281	14
2.040	125	140	10.5
2.540	102	104	9.5
2.963	79	82.4	8.6

At small times the relative uncertainty arising from statistical fluctuation as shown by the standard deviation is small and it does not show clearly on a graph of reasonable scale. Table I is, therefore, included to indicate the degree of agreement between the observations and the calculations based on the exponential distribution of exponential populations. It is seen that the agreement is excellent.

The detail concerning the individuals surviving to very long times is shown in Fig. 1. It is seen that with few brief exceptions, the number

of observed survivors falls within one standard deviation of the calculated curve.

Time intervals were chosen which divided the number of observations into twelve approximately equal groups. For these times the expected numbers were obtained from calculations based on Eq. 7. The χ^2 test was then applied to the differences between the calculated and observed numbers. The conclusion reached was that differences as great as those observed would be expected to arise from statistical fluctuations in one out of four such sets of data.

The observations reported on the survival of arcs are believed to illustrate the exponential distribution of exponential populations. The importance of the distribution is emphasized by the fact that we have obtained repeated sets in which a similar excess of observations at very short and at very long times has appeared. The calculations which we have made will be useful in specifying the statistics of any population with properties lying anywhere in the range from the simple exponential to the exponential distribution of exponential populations.

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THE CHARACTERISTICS OF AN ELECTROMAGNETIC WAVE REFLECTED FROM A MOVING OBJECT

BY

C. F. COLE, JR.[1]

ABSTRACT

A theoretical study is carried out to investigate the difference in the properties of an electromagnetic wave reflected from a fixed object and a moving object. Solutions of a wave equation in rectangular coordinates are used with different boundary conditions for the determination of the reflected wave characteristics.

The variation in the values of angle of incidence and reflection with time, as an object moves, are shown to be related to Doppler frequency as well as the relative velocity. Expressions for an electromagnetic wave reflected from both the fixed and moving object are given. A relationship between relative velocity with respect to the source of incident wave and the reflected wave frequency and amplitude is described.

INTRODUCTION

It is well known that the characteristics of a plane wave may be determined by the application of boundary conditions to wave equations. For an electromagnetic wave, Maxwell's equations for a given coordinate system are used to establish the action of time changing electric and magnetic fields. The time variation of the fields and the boundary conditions establish the solutions to the wave equation. In this paper a solution of a wave equation will be sought which will establish the properties of an electromagnetic wave reflected from a moving object. This reflected wave will be shown to decrease in amplitude as the moving object increases its relative velocity with respect to the source of the incident wave. Another characteristic of the reflected wave is that the wave shows frequency deviation from that of the incident wave. This frequency deviation is the same as the Doppler frequency and varies with the relative velocity of the moving object. The reflected wave from the moving object possesses all of the properties of a phase modulated wave.

An analysis of the way in which a wave is reflected from both a fixed and moving object is given below with the knowledge that many papers have been written on the subject of reflection or scattering of electromagnetic waves from stationary bodies (**1**, **2**).[2] A great deal of this material has been devoted to the study of the scattering properties for various geometrical configurations (**3**, **4**). However, an analysis of scattering from moving objects has not received this much attention.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

It has been known for many years that movement of an object in the path of an electromagnetic wave of the source will cause a Doppler effect to be detected by the observer of the electromagnetic radiation (5). What is the difference between an electromagnetic wave reflected from a fixed object and the same wave relected from a moving object? The difference may be found in the solution of Maxwell's wave equation for two different boundary conditions. The wave equation given below will be the basis for obtaining a solution. This wave equation is for a variable electric field intensity in a homogeneous isotropic medium. The equation is

$$\frac{\partial^2 E}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \qquad \begin{matrix} H_x = 0, & H_y = 0 \\ E_y = 0, & E_z = 0. \end{matrix} \tag{1}$$

In this equation, E is the electric field intensity vector in the plane of polarization, c is the velocity of light in free space, y is the coordinate along which the plane wave is traveling and t is time.

The solution of the above wave equation will be given for the conditions for a fixed object and for a moving object with a sine wave variation is the electric field intensity.

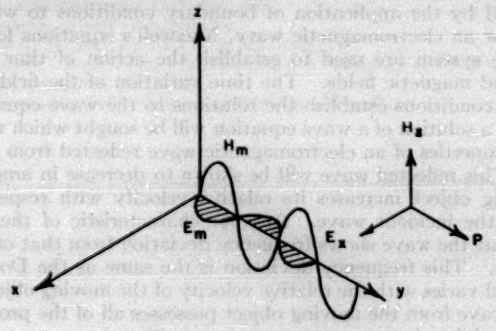

FIG. 1. A traveling wave.

REFLECTION FROM A FIXED OBJECT

For an incident wave, whose time variation of amplitude of the electric field intensity is sine wave function, the solution is

$$E_z(y, t) = A_i e^{j\left(\omega t - \frac{\omega_i y}{c}\right)}, \tag{2}$$

where: y is a point in space to which the wave has traveled in a time, t; ω_i/c is equal to β and is known as the phase constant; ω_i is the angular velocity of the traveling wave; and A_i is the amplitude of the wave from

a source of incident radiation. The wave is assumed to be traveling to the right along the y-axis as shown in Fig. 1.

If the wave expressed by Eq. 2 is reflected from a fixed object as shown in Fig. 2, then a new solution to Eq. 1 is necessary. This solution is found in almost any book dealing with wave propagation (**6**, **7**). The method of analysis in solving the equation will be a parallel to that of Jordan (**8**). Jordan's work is carried out in vector form.

Equation 2 may be rewritten in terms of a unit vector, N, and a radius vector, $\mathbf{r}$. Here the distance given by the radius vector is the distance between the origin and the plane of constant phase. A vector form of Eq. 2 may be written which will show a vector wave front traveling in a direction normal to the plane of constant phase. The equation for the plane wave is given as

$$E_z = A_i e^{j\omega\left(t+\frac{\mathbf{N}\cdot\mathbf{r}}{c}\right)}, \tag{3}$$

in which E_z is the electric field intensity vector in the plane of polarization, $\mathbf{N}$ is a unit vector normal to the plane of constant phase, $\mathbf{r}$ is a radius vector giving the distance to the plane of constant phase, and A_i is the amplitude constant which will be maximum in value at the source of radiation. Again, c is the velocity of light in free space.

If the dot product of the vector $\mathbf{N}$ and the vector $\mathbf{r}$ are expressed by their direction cosines directed along the rectangular coordinate axis, the equation for the incident wave is

$$E_z(y, t) = A_i e^{j\omega\left(t+\frac{lx+my+nz}{c}\right)}. \tag{4}$$

In this equation l, m, and n are direction cosines normal to the wave front. These are found from Fig. 2 in terms of the angle of incidence.

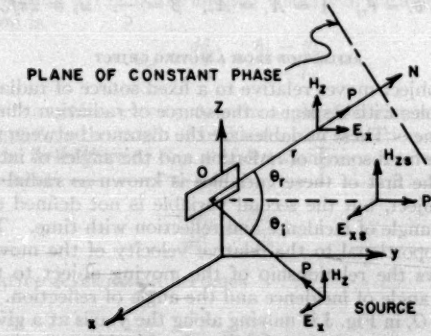

FIG. 2. A wave reflected from a fixed object.

The angle of incidence is assumed to equal the angle of reflection. The direction cosines are written as

$$l = \cos\theta_i, \quad m = -\sin\theta_i, \quad n = 0$$

which is for the incident wave going toward the fixed object. n is zero as there is no normal component of a wave traveling along the z-axis. So Eq. 4 now becomes

$$E_{xi}(y, t) = A_i e^{j\omega_i\left(t + \frac{x\cos\theta_i - y\sin\theta_i}{c}\right)}. \tag{5}$$

The same steps may be taken for the reflected wave coming from the fixed object. The direction cosines along the coordinate axis are rotated by 180° to those of the incident wave. Listing the direction cosines in order as to location on axis, these are

$$l = \cos\theta_r, \quad m = \cos(\pi/2 - \theta_r) = \sin\theta_r, \quad n = 0.$$

Substituting the direction cosines into Eq. 4, the equation for the reflected wave becomes

$$E_{xr}(y, t) = A_r e^{j\omega_i\left(t - \frac{(x\cos\theta_r + y\sin\theta_r)}{c}\right)}. \tag{6}$$

A_r is the amplitude of the reflected wave at the origin of reflection.

If we take the sum of Eqs. 6 and 5, then we have the equation for a standing wave to the right along the y-axis due to the interference of the reflected wave. The solution of the wave Eq. 1 for a fixed object is then

$$E_x(y, t) = 2A\cosh(j\beta x\cos\theta)e^{j(\omega_i t - \beta y\sin\theta)}$$

$$\theta = \theta_i = \theta_r, \quad A = A_i = A_r, \quad \beta = \frac{\omega_i}{c}, \quad \omega_i = 2\pi f_i. \tag{7}$$

REFLECTION FROM A MOVING OBJECT

When an object moves relative to a fixed source of radiation, there are two variables with respect to the source of radiation that vary as a function of time. These variables are the distance between the moving object and the fixed source of radiation and the angles of incidence and reflection. The first of these variables is known as radial velocity of the moving object, but the second variable is not defined except as a change in the angle of incidence and reflection with time. This second variable is proportional to the relative velocity of the moving object. Figure 3 shows the relationship of the moving object to the rate of change of the angle of incidence and the angle of reflection.

An object, O, in Fig. 3 is moving along the y-axis at a given relative velocity, the angular velocity of the angle of incidence and the angle of

reflection becomes proportional to this relative velocity. In Fig. 3 the relative velocity of the moving object is given by

$$v_R = \frac{dy}{dt} \doteq \frac{y_1 - y_0}{t_1 - t_0}, \tag{8}$$

where: t_0 = the time for the object at the position O; y_0 is the position of the object on the y-axis at the time, t_0; t_1 is the time for the object at position O'; y_1 is the position of the object at the time, t_1.

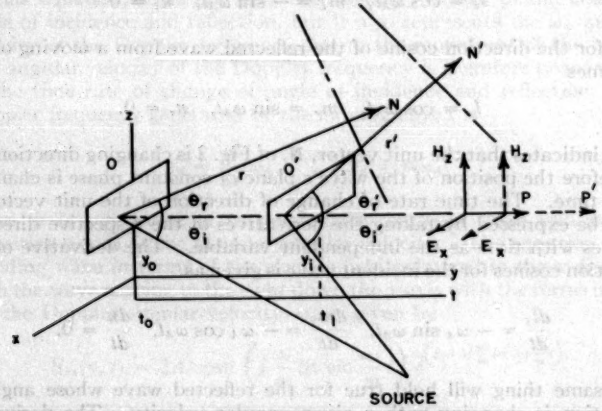

FIG. 3. A wave reflected from a moving object.

Again referring to Fig. 3, the angular velocity of the angle of incidence and reflection is

$$\omega_A = \frac{d\theta_{ir}}{dt} \doteq \frac{\theta_1 - \theta_0}{t_1 - t_0}. \tag{9}$$

Here, ω_A is the angular velocity, θ_0 is the angle of incidence and reflection at time t_0 and θ_1 is the angle of incidence and reflection at the time t_1. If we take the derivative of Eqs. 8 and 9 with respect to times, then the angular acceleration is

$$\frac{d^2\theta_{ir}}{dt^2} \doteq \frac{d\omega_A}{dt} \tag{10}$$

and the relative acceleration becomes

$$\frac{d^2y}{dt^2} \doteq \frac{dv_r}{dt}. \tag{11}$$

From Eq. 8 the distance along the y-axis is $y = v_R t$, and the value of the angle of incidence and reflection is found also from the relationship of Eq. 9 which is $\theta_{ir} = \omega_A t$. θ_{ir} would therefore represent the fixed angle of incidence and reflection if the object were fixed.

If the angular function, $\omega_A t$, is substituted for the fixed angles θ_i and θ_r which will be equal to θ_{ir} as the angle of incidence and reflection are equal, then the direction cosines of vector dot product for the incident wave become

$$l_i = \cos \omega_A t, \quad m_i = -\sin \omega_A t, \quad n_i = 0$$

and for the direction cosine of the reflected wave from a moving object becomes

$$l_r = \cos \omega_A t, \quad m_r = \sin \omega_A t, \quad n_r = 0.$$

This indicates that the unit vector, **N**, of Fig. 2 is changing direction and therefore the position of the wave's plane of constant phase is changing with time. The time rate of change of direction of the unit vector **N**, may be expressed by taking the derivatives of the respective direction cosines with time as the independent variable. The derivative of the direction cosines for the incident wave is given as

$$\frac{dl_i}{dt} = -\omega_A \sin \omega_A t, \quad \frac{dm_i}{dt} = -\omega_A \cos \omega_A t, \quad \frac{dn}{dt} = 0. \tag{12}$$

The same thing will hold true for the reflected wave whose angle of reflection is changing with a given angular velocity. The derivative for the direction cosine of the reflected wave is

$$\frac{dl_r}{dt} = -\omega_A \sin \omega_A t, \quad \frac{dm_r}{dt} = \omega_A \cos \omega_A t, \quad \frac{dn_r}{dt} = 0. \tag{13}$$

Rewriting Eqs. 5 and 6 in terms of these time changing direction cosines, the equation for the incident wave becomes

$$E_{xi}(y, t) = A_i e^{j\omega_i \left(t - \frac{\omega_A x \sin \omega_A t - \omega_A y \cos \omega_A t}{c}\right)} \tag{14}$$

and that for the reflected wave becomes

$$E_{xr}(y, t) = A_r e^{j\omega_r \left(t - \frac{(-\omega_A x \sin \omega_A t + \omega_A y \cos \omega_A t)}{c}\right)} \tag{15}$$

$$\beta = \frac{\omega_i}{c} \qquad \omega_i = 2\pi f_i.$$

Again, A_i and A_r are the amplitudes of the incident and reflected waves from the moving object. These amplitudes are assumed equal as the perfect reflector absorbs no energy from the incident wave.

By taking the sum of Eqs. 14 and 15, the equation of the standing wave seen at the right of the moving object due to the incident and the reflected waves will be

$$E_{xs}(y, t) = 2A \cosh (j\omega_A \beta x \sin \omega_A t) e^{(j\omega_i t - j\omega_A \beta y \cos \omega_A t)}. \tag{16}$$

In this equation, ω_A is not only the angular velocity of the changing angle of incidence and reflection, but it also represents the ω_d, angular velocity of the Doppler frequency due to the movement of the object. The angular velocity of the Doppler frequency is therefore proportional to the time rate of change of angle of incidence and reflection. The Doppler frequency generated by the moving object is

$$f_d = \frac{m}{2\pi} \omega_A, \qquad \omega_d = m\omega_A. \tag{17}$$

Here, m is a constant of proportionality between these two angular velocities, and ω_d is the Doppler angular velocity. The equation of the standing wave in terms of the electric field polarized in the x-direction with the wave moving to the right down the y-axis with the terms including the Doppler angular velocity, ω_d, is given by

$$E_{xs}(y, t) = 2A \cosh \left(j \frac{\omega_d}{m} \beta x \sin \frac{\omega_d}{m} t \right) e^{\left(j\omega_i t - j \frac{\omega_d}{m} \beta y \cos \frac{\omega_d}{m} t \right)}. \tag{18}$$

If we expand term, $e^{-j \frac{\omega_d}{m} \beta y \cos \frac{\omega_d}{m} t}$, into a series of Bessel functions, the term becomes a new expression which is

$$\begin{aligned} e^{-j \frac{\omega_d}{m} \beta y \cos \frac{\omega_d}{m} t} = {} & J_0 \left(\frac{\omega_d}{m} \beta y \right) - 2 \left(J_2 \left(\frac{\omega_d}{m} \beta y \right) \cos 2 \frac{\omega_d t}{m} \right. \\ & - J_4 \left(\frac{\omega_d}{m} \beta y \right) \cos 4 \frac{\omega_d t}{m} + \cdots \Big) - j2 \left(J_1 \left(\frac{\omega_d}{m} \beta y \right) \right. \\ & \left. \times \cos \frac{\omega_d t}{m} - J_3 \left(\frac{\omega_d}{m} \beta y \right) \cos 3 \frac{\omega_d}{m} t \cdots \right). \end{aligned} \tag{19}$$

Again representing another term associated with Eq. 19, the term being

$$e^{j\omega_i t} = \cos \omega_i t + j \sin \omega_i t \tag{20}$$

which will be multipled term by term of Eq. 19 giving the product of the exponential functions of time as

$$J_0\left(\frac{\omega_d}{m}\beta y\right)(\cos\omega_i t + j\sin\omega_i t) - 2\left(J_2\left(\frac{\omega_d}{m}\beta y\right)\cos\omega_i t\cos 2\frac{\omega_d}{m}t - J_4\left(\frac{\omega_d}{m}\beta y\right)\cos\omega_i t\cos 4\frac{\omega_d}{m}t + \cdots\right) - j2\left(J_1\left(\frac{\omega_d}{m}\beta y\right) \times \cos\omega_i t\cos\frac{\omega_d}{m}t - J_3\left(\frac{\omega_d}{m}\beta y\right)\cos\omega_i t\cos 3\frac{\omega_d}{m}t + \cdots\right) - j2\left(J_2\left(\frac{\omega_d}{m}\beta y\right)\sin\omega_i t\cos\frac{\omega_d}{m}t - J_4\left(\frac{\omega_d}{m}\beta y\right)\sin\omega_i t \times \cos 4\frac{\omega_d}{m}t + \cdots\right) - 2\left(J_1\left(\frac{\omega_d}{m}\beta y\right)\sin\omega_i t\cos\frac{\omega_d}{m}t - J_3\left(\frac{\omega_d}{m}\beta y\right)\sin\omega_i t\cos 3\frac{\omega_d}{m}t + \cdots\right). \quad (21)$$

With the substitution of the proper trigonometric identities for the product of two trigonometric functions with different angles, we obtain the final equation of the standing wave along the y-axis to the right of the origin. This equation is therefore

$$E_{z_0}(y,t) = \left(2A\cosh\left(j\frac{\omega_d}{m}\beta x\sin\frac{\omega_d}{m}t\right)\right)\left[\left(J_0\left(\frac{\omega_d}{m}\beta y\right)\cdot e^{j\omega t}\right) - \left(J_1\left(\frac{\omega_d}{m}\beta y\right)\left[\sin\left(\omega_i - \frac{\omega_d}{m}\right)t + \sin\left(\omega_i + \frac{\omega_d}{m}\right)t\right] - J_3\left(\frac{\omega_d}{m}\beta y\right)\left[\sin\left(\omega_i - 3\frac{\omega_d}{m}\right)t + \sin\left(\omega_i + 3\frac{\omega_d}{m}\right)t\right] + \cdots\right) - j\left(J_1\left(\frac{\omega_d}{m}\beta y\right)\left[\cos\left(\omega_i - \frac{\omega_d}{m}\right)t + \cos\left(\omega_i + \frac{\omega_d}{m}\right)t\right] - J_3\left(\frac{\omega_d}{m}\beta y\right)\left[\cos\left(\omega_i - 3\frac{\omega_d}{m}\right)t + \cos\left(\omega_i + 3\frac{\omega_d}{m}\right)t\right] + \cdots\right) - \left(J_2\left(\frac{\omega_d}{m}\beta y\right)\left[\cos\left(\omega_i - 2\frac{\omega_d}{m}\right)t + \cos\left(\omega_i + 2\frac{\omega_d}{m}\right)t\right] - J_4\left(\frac{\omega_d}{m}\beta y\right)\left[\cos\left(\omega_i - 4\frac{\omega_d}{m}\right)t + \cos\left(\omega_i + 4\frac{\omega_d}{m}\right)t\right] + \cdots\right) - j\left(J_2\left(\frac{\omega_d}{m}\beta y\left[\sin\left(\omega_i - 2\frac{\omega_d}{m}\right)t + \sin\left(\omega_i + 2\frac{\omega_d}{m}\right)t\right]\right. - J_4\left(\frac{\omega_d}{m}\beta y\right)\left[\sin\left(\omega_i - 4\frac{\omega_d}{m}\right)t + \sin\left(\omega_i + 4\frac{\omega_d}{m}\right)t\right] + \cdots\Big)\Big]. \quad (22)$$

An examination of Eq. 22 indicates that the amplitude of the standing wave will approach zero as the Doppler frequency (ω_d) increases without limit (9). Since the Doppler frequency is proportional to the relative velocity, it would be possible to have a standing wave of zero amplitude if the velocity of the moving object were great enough. The terms of Eq. 22 containing trigonometric functions shows that the standing wave outward from the surface of a moving object contains the Doppler frequency as a frequency deviation from the frequency of the incident wave. Equation 22 is therefore a representation of a phase modulated wave whose amplitude and frequency deviation are dependent upon the relative velocity of the object with respect to the source of the incident electromagnetic wave.

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Gas-Turbine Engine for Ships.—A detailed description of the control system that has made a converted Liberty ship "more maneuverable than any other ship of its size in existence" was offered recently at a conference in Washington, D. C., sponsored by The American Society of Mechanical Engineers.

The ship, the *John Sergeant*, is powered by gas turbine engines, instead of the conventional steam turbine used in most sea going vessels.

According to F. H. Van Nest, Manager, Design Engineering Gas Turbine Department, General Electric Company, the low fuel consumption of the ship is "more than competitive with the best steamships in its power range, and the reduced annual operating cost resulting from less engine room personnel makes this type installation competitive even in the higher power ranges where the steam turbine driven vessel normally excells." His paper was delivered before the Third Annual Gas Turbine Conference and Exhibit at the Hotel Shoreham.

Mr. Van Nest attributed the success and maneuverability to the combination of the gas turbine with a reversible and controlled pitch propellor. Controlled pitch propellors, used on aircraft for some time, permit the angle of the blades to be changed, thereby increasing or decreasing "pull" or "push" developed.

In his paper, Mr. Van Nest described some of the complex engineering problems that had to be solved in order to guarantee safe, reliable operation of the gas turbine unit.

Emergency Rescue Container.—"That others may live" will take on a literal meaning for supersonic jet pilots assigned to the search and rescue of downed airmen at sea or in inaccessible areas—both in tropic and arctic zones. The motto is being associated with a new emergency rescue container developed by the Air Research and Development Command. The container packs 450 lb. of survival equipment.

Measuring 23 in. in diameter, the 14-ft. long shell hooks onto the bomb rack of supersonic jet aircraft. Equipment within the container enables an airman to survive until actual rescue is effected. It is paradropped to those in distress.

Each container is crammed with enough food and medical supplies to last one man 30 days. Both arctic and tropic gear are available. Standard rescue equipment such as signal flares, bedding, clothing, matches, tools, a gun, knife and radio are stowed into the container.

Dropping the container to downed airmen at sea or in areas not easily reached on land is as simple as jettisoning a bomb.

When a pilot approaches his mercy mission area, he merely triggers the bomb-release mechanism. The container falls free as an automatic timing device releases its nose cone. As the cone plummets to earth a parachute billows out and the rest of the container floats gently down toward survivors.

For sea rescues, survival equipment is packed into a raft. The raft is automatically inflated in the air by a carbon dioxide bottle which is activated when the container's nose cone is released.

Now undergoing final tests at ARDC's Wright Air Development Center, Dayton, Ohio, the container was developed under Air Force contract with Beech Aircraft Corp., Wichita, Kansas.

Howard F. Holton, of WADC's Aeronautical Accessories Laboratory acted as project engineer for the emergency rescue container.

THE NATURE OF THE PHOTOSENSITIVITY OF GEIGER COUNTERS *

BY

H. O. ALBRECHT [1] AND C. E. MANDEVILLE [1]

ABSTRACT

Some aspects of the photosensitivity of the Geiger counter are discussed. Methods for increasing response to ultraviolet and near ultraviolet photons are described. The photosensitivity is traced to formation of a surface film upon the inner wall of the cathode; that is, a quasi-metallic alkyl coating is assumed to be formed. Relatively short wave radiation of sufficient energy to form free radicals in hydrogen and organic quenching gases is required to increase ("activate") the sensitivity.

I. INTRODUCTION

GM counters were early used as sensitive photon detectors in searching for energetic ultraviolet radiation from chemical reactions, including those occurring in living cells (1).[2]

Ultraviolet sensitive photon counters have more recently been employed in studies of the photostimulated ultraviolet emission of excited phosphors (2, 3). These photon counters were especially convenient for this purpose because of their strongly selective response to the ultraviolet region (2700 Å) and a low response to the photostimulating visible source.

Experiments of an earlier date (4) had shown that increased ultraviolet sensitivities could be obtained by subjecting self-quenching counters to an electrical discharge while cooled in liquid nitrogen. Subsequently, it was observed by the present writers that the mere irradiation of the counter cathode with ultraviolet of wave length 2537 Å could, with certain gas fillings, increase the photosensitivity. In this latter treatment, no potential difference was applied to the electrodes of the counter. It has also been noted that a d-c. corona discharge at room temperature will increase the photosensitivity of counters when the filling gases are not decomposed in the gross sense. This "activation" or enhancement of photosensitivity by these means has varied considerably, from counter to counter, despite seemingly identical conditions of treatment and counter construction.

An attempt to stabilize and increase the effects of such activating treatments was an objective of the present effort, as was the attainment of a clear understanding of the various curious effects. This investiga-

* Assisted by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

tion inevitably involved the study of the counter discharge itself under various conditions of filling gas and cathode material.

The construction of the counter as used for most comparisons is shown in Fig. 1. The essential features are an ultraviolet transmitting envelope of Corning 9741 glass, a metallic gauze cathode to admit light,

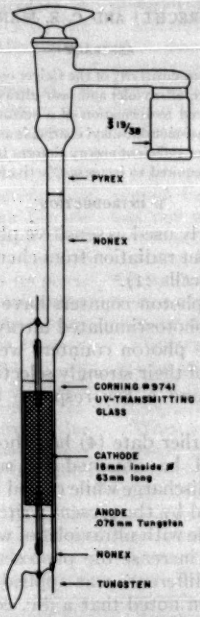

FIG. 1. Photon counter designed for rapid changes of the filling gas.

and a ground joint connection to the filling system. The lubricant used was Apiezon N grease. The gauze cathodes of the counters were usually of copper, but platinum, brass, zinc, and "electrically conducting" glass were also employed, the latter cathode material being prepared by the Corning Glass Works as a conducting layer on the inside of Corning 9741 glass. The anode was usually a tungsten wire of diameter 0.096 mm., though platinum and platinum clad tungsten were also used. Few if any of the aberrant phenomena observed could be ascribed to variations in material or condition of the anodes.

Routine cleaning procedure for counters with copper cathodes commenced by introducing into the counter tube a mixture of 1 volume of 30 per cent hydrogen peroxide in nine volumes of glacial acetic acid.

When the cathode surface was etched to brightness by formation of $Cu(CH_3COO)_2$, it was rinsed repeatedly by addition of distilled water. A similar procedure was usually followed for all metals, not necessarily forming the acetate, to remove impurities. Occasionally, counters containing platinum and gold cathodes were cleaned by adding nitric acid, then hydrofluoric acid, and rinsing. Drying was accomplished by opening the tubulature at the bottom of the counter shown in Fig. 1, heating to 400° C., while passing hydrogen through the counter. After sealing off the tubulature, the counter was attached to the filling system and again heated to 400° C. in hydrogen. The use of hydrochloric and sulfuric acids was avoided at all times.

It should perhaps be remarked that most photoactivated counters lost about half of their ultraviolet induced photosensitivities for ultraviolet and near visible radiation within 24 hours after activation.

II. EXPERIMENTAL RESULTS

Results, typical of the treatment of a counter, are shown in Fig. 2. The response of the counter to standard light sources is plotted after

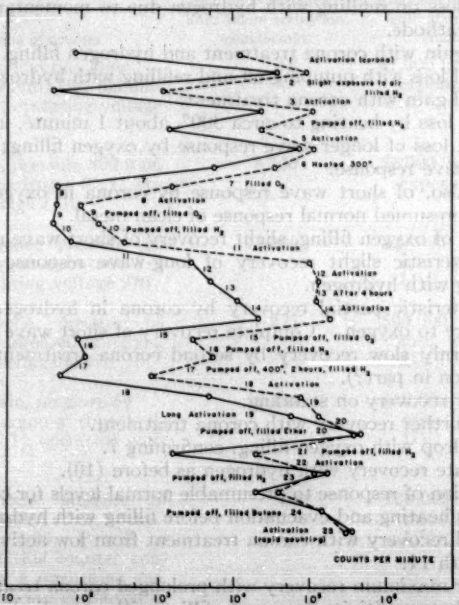

FIG. 2. Response of copper gauze photon counter under various conditions of treatment. Successive operations are represented by constant lengths along the axis of ordinates.

each phase of the operations. In the case of these particular data, the ultraviolet for activation was generated by a d-c. corona discharge at room temperature which was passed through the counter tube under conditions of normal polarity, that is, the wire of the counter was positive, and the metallic gauze cylinder was negative. In the example under consideration, a copper counter was subjected to various treatments and fillings as indicated by the comments shown on the drawing of Fig. 2. The broken line gives response to 2537 Å, and the unbroken one to a 75-watt incandescent tungsten lamp, both at fixed distances. The counting rates at the ends of each segment are those before and after each treatment. Thus, just before the fifteenth treatment, the response to light at 2537 Å was 10^5 counts/min, and after the counter tube was pumped out and filled with oxygen, the counting rate was only about 2×10^3 counts/min. A detailed description of the effects of each step of the procedures of Fig. 2 is given below.

1. Gain by corona treatment at room temperature starting from clean condition of cathode (hydrogen filling).
2. Large loss on refilling with hydrogen due to momentary access of air to cathode.
3. Usual gain with corona treatment and hydrogen filling.
4. Normal loss with pumping off and refilling with hydrogen.
5. Normal gain with corona treatment.
6. Partial loss by heating to circa 300°, about 1 minute, in hydrogen.
7. Abrupt loss of longer wave response by oxygen filling; less loss of short-wave response.
8. Loss, also, of short wave response by corona in oxygen, to level below presumed normal response of clean metal.
9. Repeat of oxygen filling, slight recovery of short-wave response.
10. Characteristic slight recovery of long-wave response by merely refilling with hydrogen.
11. Characteristic partial recovery by corona in hydrogen after full exposure to oxygen. Complete recovery of short wave response.
12. Stubbornly slow recovery by second corona treatment (chemical reduction in part?).
13. Further recovery on standing.
14. Some further recovery with corona treatment.
15. Sharp drop with oxygen filling, confirming 7.
16. Moderate recovery with hydrogen as before (10).
17. Reduction of response to presumable normal levels for clean copper by long heating and evacuation before filling with hydrogen.
18. Normal recovery with corona treatment from low activity. Compare with 11.
19. Further maximum recovery with prolonged corona treatment.
20. Peak photosensitivity by merely filling with pure diethyl ether.

21. Drop to normal level by filling with hydrogen. Compare with 4.
22. Abnormally high recovery with corona in hydrogen, following ether fillings.
23. Abnormally slight loss, refilled with hydrogen.
24. Gain with filling with pure butane, but less than with ether.
25. Gain by rapid counting in butane, instead of corona treatment.

A wide variety of counters (cathodes of gold, platinum, etc.) were subjected to a sequence of events much like those described in Fig. 2; the activating agent was also varied, being for some sequences a discharge at liquid nitrogen temperatures, for others, a discharge at room temperature, and for still others irradiation by an ozone bulb at 2537 Å (potential removed from the electrodes of the counters during this last activation procedure). In all instances, the results were qualitatively the same as those depicted in Fig. 2.

TABLE I.

Description of counter	Response to incandescent bulb before activation, counts/min.	Response to incandescent bulb after activation, counts/min.
Electrically conducting glass, pressure 60 mm. Hg, helium-ether (5:1), operating voltage 1000	18	47,000 (after 25-min. exposure to UV)
Gold cathode, pressure 300 mm. Hg, argon-hydrogen (9:1), operating voltage 1310	8,700	52,000 (after 1-min. exposure to UV)
Copper cathode, pressure 60 mm. Hg, argon-isobutane (5:1), operating voltage 920	13,000	500,000 (after 45-min. exposure to UV)
Copper cathode, pressure 60 mm. Hg, argon-ether (5:1), operating voltage 925	227	500,000 (after 12-min. exposure to UV)
Copper cathode, pressure 59 mm. Hg, argon-*n* (butyl alcohol) (6.4:1), operating voltage 750	10,400	620,000 (after 54-min. exposure to UV)
Copper cathode, pressure 20 mm. Hg, diethyl ether, operating voltage 980. It will be noted that this counter contained only a quenching gas. No inert gas was employed in addition.	960	220,000 (after 14-min. exposure to UV)

In the course of these measurements, about 400 counter fillings were tested to study the effects of past history and for confirmatory purposes. Among these were 46 different combinations of gases and four frequently used cathode materials. Pressures varied from about 20 mm. Hg for counters filled with a quenching gas only to 60–100 mm. Hg for common mixtures and nearly one atmosphere for counters filled only with a noble gas. Operating voltages were in the range of 500 to 2000 volts, frequently lying between 800 and 1300 volts. About fifty counters of distinctly different behavior (pulse shape, plateau length, photosensitivity, etc.) were tested.

Table I gives results indicating a sharp change in photosensitivity after irradiation with light at 2537 Å from an ozone bulb.

The experimental data which have been cited in this section will be interpreted in the text to follow.

III. THE PHOTOSENSITIVE FILM

A. Evidence for the Presence of a Surface Film

In considering indications of the presence of a surface layer, it should be first pointed out that the gas fillings are not themselves the source of the photosensitivity. In most instances the incident wave lengths were less than 2537 Å (4.85 eV); in fact, the data of Table I concern response to wave lengths greater than 3000 Å. These frequencies correspond to far less energy than are the ionization potentials of such elements as H, He, C, O, and A. Thus, it is logical to conclude that the photosensitivity under study resides at the metallic surface of the cathode where the work function is at most a few electron volts.

A series of observed facts are consistent with the assumption that the photosensitivity lies in the surface layer:

1. After activation of a filled counter by ultraviolet illumination and subsequent removal of the filling, prolonged pumping (over a period of hours) was found to reduce the photosensitivity by a factor of ten. A pumping time of five minutes reduces the photoelectric response by a factor of about two.

2. With continued activation, the maximum of photoelectric response is observed to shift toward longer wave lengths.

3. Counters containing only inert gases and the very cleanest metallic cathodes never provided a response rivalling that of counters containing in addition hydrogen or an organic quenching gas.

4. Heating of filled photoactivated counters resulted in a loss of sensitivity, suggesting that an adsorbed surface layer might have been driven off. The temperature of heating was maintained below that required for decomposition of the gases involved.

5. Films are also formed on the cathode when gases which are definitely unfavorable for photosensitivity are used (oxygen, and some oxygen, sulfur and halogen-containing compounds). Irradiation with ultraviolet or corona treatment likewise speeds the formation of these films, and the sensitivity, even to 2537 Å, may *fall* to practically zero. The greater chemical reactivity of such gases under the conditions of ultraviolet exposure or corona is a ready explanation of this effect. When a counter thus "deactivated" is refilled with a gas favorable for photosensitivity, more "activation" than normal is required to regain a given photosensitivity. Indeed, in the extreme case it may not be possible to attain any sensitivity without a thorough cleaning of the cathode.

B. Formation of the Surface Film

The dissociation energies of the compounds of the various gas fillings employed in these investigations are less than the energy of photons at 2537 Å. Thus, active radicals, groups of atoms having a net unsatisfied valence, can be liberated by the activating ultraviolet to form a deposition upon the inner wall of the cathode of the counter (5). It is entirely possible that these radicals may form a quasi-stable compound with atoms of copper, say, at the surface of the cathode so that a film of reduced work function results. As an example of this type of behavior, the attachment of a methyl radical to copper might be considered. An equilibrium condition of the metal-alkyl compound could result. Thus,

$$\underbrace{\mathrm{Cu:\overset{\overset{\mathrm{H}}{\vdots}}{\underset{\underset{\mathrm{H}}{\vdots}}{\mathrm{C}}}:H}}_{\text{co-valent form}} \rightleftarrows \underbrace{\mathrm{Cu}^{\oplus}\left[\overset{\overset{\mathrm{H}}{\vdots}}{:\underset{\underset{\mathrm{H}}{\vdots}}{\mathrm{C}}:H}\right]^{\ominus}}_{\text{ionic form}}$$

In its ionic form, the electronic binding energy is probably reduced so that any response to long wave light is increased. An electron is released according to the following diagram.

$$\mathrm{Cu}^{\oplus}\left[\overset{\overset{\mathrm{H}}{\vdots}}{:\underset{\underset{\mathrm{H}}{\vdots}}{\mathrm{C}}:H}\right]^{\ominus} \xrightarrow{h\nu} \mathrm{Cu} + \left[\overset{\overset{\mathrm{H}}{\vdots}}{\underset{\underset{\mathrm{H}}{\vdots}}{\mathrm{C}}:H}\right]^{\oplus} + e^{\ominus}$$

This electron produces a count, and the positively charged radical is neutralized on the cathode to reform the cuprous-alkyl film. A positively charged alkyl radical is plausibly more stable than a negative one, an additional reason for assuming this cyclic process to yield a lower

work function than copper. Of course, dissociation can also occur into saturated and unsaturated compounds having no affinity for metals. For example, C_4H_{10} can dissociate into C_2H_6 (saturated) and C_2H_4 (unsaturated).

IV. DISCUSSION

The foregoing observations are consistent with effects long observed in photon counters. For example, with continued use, it has always been noted that counters filled with hydrogen or containing organic quench gases show a steadily increasing photosensitivity to near ultraviolet radiation. These experiments concerning ultraviolet "activation" of photosensitive surfaces suggest that the short wave radiation in the counter discharge itself leads to the formation of a surface film.

Spurious counts in Geiger counters are usually assigned to any one of several causes (6) such as:

1. De-excitation of metastable states of atoms excited in the gaseous discharge;
2. Electron attachment in the discharge to form heavy negative ions and the subsequent release of the electron to imitate another count; and
3. Release of electrons from the cathode during positive ion bombardment.

Two additional sources of spurious counts are developed by the ultraviolet activation process:

4. Static charges may be generated by ultraviolet irradiation, and ions therefrom may at times be released into the counting volume; and
5. "After-emission" from the photosensitive film.

Spurious counts may also develop from the structural features of photon counters. For example, if the openings in the gauze cathode of the counter of Fig. 1 are too large, positive ions may collect upon the inner glass wall to be subsequently attracted by the cathode, yielding erratic counter operation.

With regard to "after-emission" Seeger (7) and others have suggested that electrons might be trapped in surface films in a manner analogous to electron storage in phosphors. Thermal release could then give rise to spurious counts. During the course of the present investigations, very high spurious counting rates were often noted immediately following activation. This effect decays with time, as the shallow traps are emptied.

The studies cited in this paper suggest several possible uses for the Geiger counter, one being that it might serve as a sensitive device for investigations of the physics of surfaces. These data also suggest the

possibility of developing an ultrasensitive photon detector by exposing a counter to intense pulses of activating ultraviolet with voltage removed from the electrodes of the counter. Between these activating pulses, the potential could be restored and the sensitized counter irradiated by any photons which it might be desired to detect.

It is apparent that a Geiger counter arrangement might also be used to detect formation of small quantities of free radicals which have been formed by exposure to radiation.

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Radioactive Waste Recovery.—Vast quantities of chemical waste containing radioactive cesium stored at the Atomic Energy Commission's Hanford plant near Richland, Wash., may some day provide for the industrial-scale irradiation of foods, drugs, plastics, rubber, chemicals and other materials.

Burton F. Judson of General Electric Company's Chemical Processing department at the Hanford plant reported on efforts to recover radioactive cesium at the 1958 Nuclear Congress. General Electric has undertaken studies of the possibility of use of the cesium at the request of the AEC.

The studies now underway at Hanford are aimed at estimating the potential market for fission products as industrial irradiation sources and developing the technology that will permit the efficient recovery of cesium from separations plant waste solutions, Judson said.

Should a strong demand be foreseen for the immediate future, an installation capable of producing large amounts of cesium-137 could be accomplished by converting existing but unused facilities at a capital cost of about $1,500,000.

At the present time the AEC is marketing cesium at $10 per curie, the radiation equivalent of one gram of radium. Judson foresees a substantial reduction in the cost of this radioisotope if the large-scale plant is put into operation.

Automatic Post Office.—Details of the world's first automated post office were made public for the first time in this country at the IRE Convention. The presentation, made by Dr. Maurice Levy, Technical Advisor to the Canadian Post Office, and his associates, covered an automated postal system now being tested in actual operation in Ottawa, Canada.

It is expected that the system will do much to reduce the time and cost of handling large volumes of mail by using electronic sorters. Present hand sorting methods are rapidly becoming impractical because mail volume is increasing faster than population growth.

The first step in the system is to separate the mail to be automated from the remainder of the mail. This is accomplished by utilizing stamps impregnated with fluorescent material. Letters so stamped are sent to an electronic printer, by which an operator codes the back of each envelope with a special code giving address information. Fluorescent ink is used for the printed code.

From this point on, the coded mail is sorted automatically on a conveyor system controlled by an electronic computer. The envelopes are routed to high speed electronic sorters, which sense the coded information and convey the letters to the proper mail containers.

Once letters reach local delivery areas the familiar mailman takes over.

Dr. Levy estimated that automation equipment of this kind for large cities would probably cost in the neighborhood of half a million dollars. The system is by far the simplest so far devised, and is suitable for cities with populations of 100,000 or more.

Special attention given to coding techniques and error checking devices has reduced the possibilities of errors to a point where less than one letter out of 10,000 will be misrouted.

Codes have been chosen so that the system can eventually be used internationally. The U. S. Post Office is presently working on an electronic mail sorting system which it is expected would use a similar coding system.

BENDING AND VIBRATION OF ELASTICALLY RESTRAINED CIRCULAR PLATES

BY

C. LAKSHMI KANTHAM [1]

SUMMARY

In the bending and vibration problems, it is found that the values of deflection and natural frequency vary considerably with change of end or edge conditions. In this paper a general boundary condition involving a linear relation between moment and slope at an edge is considered, so that the simply supported and completely fixed edge conditions form the limiting cases. The constant of proportionality is the elastic restraint factor, which is a measure of the degree of fixture, varying from zero to positive infinity. Values of deflection in the one case and natural frequency in the other have been determined for intermediate values of the elastic restraint factor.

INTRODUCTION

In the bending and vibration of plates it is found that the values of maximum deflection and natural frequencies, respectively, vary considerably from the simply supported to clamped edge conditions. For an estimation of these characteristics in the intermediate range a generalized boundary condition may be assumed, of which the simply supported and clamped edges become the limiting cases. While Bassali (1)[2] considers the ratio of edge moment to the cross-wise moment as a constant, Newmark (2), Lurie (3), Klein (4), and other investigators (5, 6) consider that moment and slope at an end are proportional in their analyses of various structures.

Here the definition of elastic restraint given by $\alpha = \beta M$, as in Timoshenko (7), is followed, where α is the slope at any edge, M the corresponding edge moment per unit length while β is the elastic restraint factor. $\beta \to 0$ and $\beta \to \infty$ represent the two limiting cases of clamped and simply supported edge conditions respectively.

I. SYMMETRICAL BENDING OF CIRCULAR PLATES WITH ELASTICALLY RESTRAINED EDGES UNDER UNIFORM PRESSURE

Let us consider a circular plate of constant thickness with its edge elastically restrained uniformly against rotation. Using the small deflection theory we can develop for such a plate, the differential equation for the deflection as (following Timoshenko (8))

$$\frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right]\right\} = \frac{q}{D}, \tag{1}$$

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[2] The boldface numbers in parentheses refer to the references appended to this paper.

where q is the pressure, D the flexural rigidity of the plate and r the radial distance from center. Integrating Eq. 1 we get

$$w = \frac{qr^4}{64D} + \frac{C_1 r^2}{4} + C_2 \log \frac{r}{a} + C_3 \tag{2}$$

where the constants of integration C_1, C_2 and C_3 are determined by the boundary conditions. The boundary conditions for zero deflection and the slope-moment relation at the edge are

$$(i) \qquad w = 0 \text{ at } r = a$$

$$(ii) \quad \left(\frac{dw}{dr}\right)_{r=0} = 0$$

$$(iii) \quad \left(\frac{dw}{dr}\right)_{r=a} = -\beta (M_r)_{r=a}.$$

For moment shown in Fig. 1, the expression in terms of curvature is

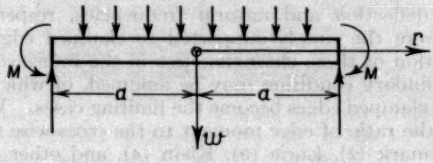

FIG. 1.

given by

$$M_r = D \left\{ \frac{d^2w}{dr^2} + \frac{\nu}{r} \frac{dw}{dr} \right\}.$$

Hence from boundary condition (iii), we get

$$\left(\frac{dw}{dr}\right)_{r=a} = -\beta D \left\{ \frac{d^2w}{dr^2} + \frac{\nu}{r} \frac{dw}{dr} \right\}_{r=a}.$$

From Eq. 2 we get

$$\left.\begin{aligned} \frac{dw}{dr} &= \frac{qr^3}{16D} + \frac{C_1}{2} r + \frac{C_2}{r} \\ \frac{d^2w}{dr^2} &= \frac{3qr^2}{16D} + \frac{C_1}{2} - \frac{C_2}{r^2} \\ \frac{1}{r} \frac{dw}{dr} &= \frac{qr^2}{16D} + \frac{C_1}{2} + \frac{C_2}{r^2} \end{aligned}\right\}. \tag{3}$$

Using boundary condition (*ii*) in the first of Eqs. 3 we see that C_2 must vanish.

Using boundary condition (*i*) in Eq. 2 we get

$$\frac{qa^4}{64D} + \frac{C_1a^2}{4} + C_3 = 0$$

or

$$C_3 = -\frac{qa^4}{64D} - \frac{C_1a^2}{4}. \tag{4}$$

From boundary condition (*iii*), we get, using Eqs. 3,

$$\frac{qa^3}{16D} + \frac{C_1}{2}a = -\beta D\left[\left(\frac{3qa^2}{16D} + \frac{C_1}{2}\right) + \nu\left(\frac{qa^2}{16D} + \frac{C_1}{2}\right)\right]$$

or

$$\frac{qa^2}{16D} + \frac{C_1}{2} = -\frac{\beta D}{a}\left[\left(\frac{3qa^2}{16D} + \frac{C_1}{2}\right) + \nu\left(\frac{qa^2}{16D} + \frac{C_1}{2}\right)\right].$$

Writing $\frac{\beta D}{a} = \lambda$ we have

$$\frac{qa^2}{16D} + \frac{C_1}{2} = -\lambda\left[\left(\frac{3qa^2}{16D} + \frac{C_1}{2}\right) + \nu\left(\frac{qa^2}{16D} + \frac{C_1}{2}\right)\right],$$

which, simplified, gives us

$$C_1 = -\frac{qa^2}{8D}\frac{(3+\nu)\lambda + 1}{(1+\nu)\lambda + 1}. \tag{5}$$

From Eq. 4 on substituting for C_1 from Eq. 5 we have

$$C_3 = -\frac{qa^4}{64D} + \frac{qa^4}{32D}\frac{(3+\nu)\lambda + 1}{(1+\nu)\lambda + 1}$$

$$= \frac{qa^4}{64D}\left[\frac{(5+\nu)\lambda + 1}{(1+\nu)\lambda + 1}\right]. \tag{6}$$

Substituting these values of C_1 and C_3 in Eq. 2, we get

$$w = \frac{qr^4}{64D} - \frac{qa^2r^2}{32D}\left[\frac{(3+\nu)\lambda + 1}{(1+\nu)\lambda + 1}\right] + \frac{qa^4}{64D}\left[\frac{(5+\nu)\lambda + 1}{(1+\nu)\lambda + 1}\right]$$

$$= \frac{qr^4}{64D} - \frac{qa^2r^2}{64D}\left[\frac{(5+\nu)\lambda + 1}{(1+\nu)\lambda + 1} + 1\right] + \frac{qa^4}{64D}\left[\frac{(5+\nu)\lambda + 1}{(1+\nu)\lambda + 1}\right].$$

Writing

$$\frac{(5+\nu)\lambda+1}{(1+\nu)\lambda+1} = N,$$

we get

$$w = \frac{qa^4}{64D}\left[\frac{r^4}{a^4} - \frac{r^2}{a^2}(N+1) + N\right]. \tag{7}$$

It is readily seen that for $\lambda \to 0$, and $\lambda \to \infty$, Eq. 7 will reduce respectively to

$$w = \frac{q}{64D}(a^2 - r^2)^2$$

$$w = \frac{q}{64D}\left[\frac{5+\nu}{1+\nu}a^2 - r^2\right](a^2 - r^2).$$

These are the well-known equations for clamped and supported circular plate, respectively.

Choosing ν as 0.3 we have

$$N = \frac{(5.3)\lambda + 1}{(1.3)\lambda + 1}.$$

From Eq. 7, w_{max} occurring at $r = 0$ is given by

$$w_{r=0} = \frac{qa^4}{64D}N.$$

The results for deflection at the center of the plate for various values of λ are presented in Table I and plotted in Fig. 2.

TABLE I.—*Variation of Deflection at the Center of a Circular Plate with Elastically Restrained Edge under Uniform Pressure.*

λ	$W_r = 0$ $\frac{qa^4}{64D}$
$\to 0$	1.00
10^{-3}	1.00
10^{-2}	1.04
10^{-1}	1.35
10^{0}	2.74
10^{1}	3.86
10^{2}	4.05
10^{3}	4.07
$\to \infty$	4.08

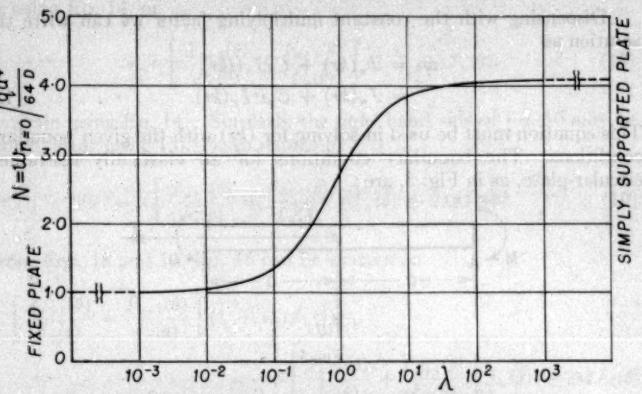

FIG. 2. Variation of deflection with elastic restraint for circular plate.

II. VIBRATION OF ELASTICALLY RESTRAINED CIRCULAR PLATES

Considering the transverse vibration of a thin elastic circular plate of constant thickness we can write the governing differential equation following Rayleigh (9) as

$$(\nabla^4 - k^4)w = 0 \tag{8}$$

with k, a characteristic parameter of vibration, given by $k^4 = \frac{p^2\rho h}{D}$ where p is the circular frequency of vibration, ρ the mass density of the material, h the thickness of the plate and D the flexural rigidity.

Now w the deflection at any point may be represented by

$$w = \sum_{n=0}^{\infty} w_n \cos n\theta \cos pk \tag{9}$$

where w_n is a function of radius r only. It is easy to see that, since w_n is independent of θ, the general term w_n must satisfy

$$\left[\frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr} - \left(\frac{n^2}{r^2} + k^2\right)\right]\left[\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr} - \left(\frac{n^2}{r^2} - k^2\right)\right]w_n = 0 \tag{10}$$

the general solution of which may be written as

$$w_n = A_nJ_n(kr) + B_nJ_n(ikr)$$

or

$$w_n = A_n[J_n(kr) + C_nJ_n(ikr)]. \tag{11}$$

Dispensing with the constant multiplying factor we can write the solution as

$$w_n = J_n(kr) + C_n J_n(ikr)$$
$$= J_n(kr) + C_n i^n I_n(kr). \tag{12}$$

This equation must be used in solving for (kr) with the given boundary conditions. The boundary conditions for an elastically restrained circular-plate, as in Fig. 3, are:

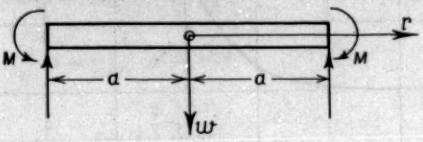

FIG. 3.

$$\left.\begin{aligned} &(i)\ w = 0 \quad \text{at} \quad r = a \\ &(ii)\ \left(\frac{dw}{dr}\right)_{r=a} = -\beta D\left\{\frac{d^2w}{dr^2} + \frac{\nu}{r}\frac{dw}{dr}\right\} \end{aligned}\right\}. \tag{13}$$

With the boundary condition (i), we have from Eq. 12

$$J_n(ka) + C_n i^n I_n(ka) = 0. \tag{14}$$

With boundary condition (ii), from Eq. 12, we get

$$k[J_n'(ka) + C_n i^n I_n'(ka)] = -\beta D[k^2\{J_n''(ka) + C_n i^n I_n''(ka)] + \frac{\nu k}{a}[J_n'(ka) + C_n i^n I_n'(ka)]. \tag{15}$$

Setting $ka = \theta$ and $\frac{\beta D}{a} = \lambda$, Eq. 15 becomes

$$J_n'(\theta) + C_n i^n I_n'(\theta) = -\lambda\theta[J_n''(\theta) + C_n i^n I_n''(\theta)] - \nu\lambda[J_n'(\theta) + C_n i^n I_n'(\theta)]$$

or

$$[J_n'(\theta) + C_n i^n I_n'(\theta)](1 + \nu\lambda) = -\lambda\theta[J_n''(\theta) + C_n i^n I_n''(\theta)]. \tag{16}$$

Making use of the relationships of derivatives of Bessel functions the left hand side of Eq. 16 may be expressed as

$$\left[\frac{n}{\theta}\{J_n(\theta) + C_n i^n I_n(\theta)\} - \{J_{n+1}(\theta) - C_n i^n I_{n+1}(\theta)\}\right](1 + \nu\lambda)$$
$$= -[J_{n+1}(\theta) - C_n i^n I_{n+1}(\theta)](1 + \nu\lambda), \tag{17}$$

by using Eq. 14, or

$$= -\left[\frac{J_{n+1}(\theta)}{J_n(\theta)} + \frac{I_{n+1}(\theta)}{I_n(\theta)}\right](1 + \nu\lambda)J_n(\theta), \tag{18}$$

by again using Eq. 14. Similarly the right hand side of Eq. 16 may be shown, after using Eq. 14, to reduce to

$$-\lambda\left[\frac{J_{n+1}(\theta)}{J_n(\theta)} + \frac{I_{n+1}(\theta)}{I_n(\theta)}\right]J_n(\theta) + 2\lambda\theta J_n(\theta). \tag{19}$$

From Eqs. 18 and 19, Eq. 16 can be written as

$$-\left[\frac{J_{n+1}(\theta)}{J_n(\theta)} + \frac{I_{n+1}(\theta)}{I_n(\theta)}\right](1 + \nu\lambda)J_n(\theta)$$

$$= -\lambda\left[\frac{J_{n+1}(\theta)}{J_n(\theta)} + \frac{I_{n+1}(\theta)}{I_n(\theta)}\right]J_n(\theta) + 2\lambda\theta J_n(\theta)$$

or

$$-\left[\frac{J_{n+1}(\theta)}{J_n(\theta)} + \frac{I_{n+1}(\theta)}{I_n(\theta)}\right]\{1 - \lambda(1 - \nu)\} = -2\lambda\theta. \tag{20}$$

This is the frequency equation from which the solutions of θ are got for different values of λ. For $\lambda \to 0$, $\lambda \to \infty$, representing clamped and simply supported edge conditions, it can be seen that Eq. 20 reduces to the well known equations:

$$\frac{J_{n+1}(\theta)}{J_n(\theta)} + \frac{I_{n+1}(\theta)}{I_n(\theta)} = 0$$

and

$$\frac{J_{n+1}(\theta)}{J_n(\theta)} + \frac{I_{n+1}(\theta)}{I_n(\theta)} = \frac{2\theta}{1 - \nu}. \tag{21}$$

Equation 20 has been solved for $n = 0$, $n = 1$ and $n = 2$ for various values of λ. The results are presented in Table II and in Fig. 4. The

TABLE II.—*Values of θ for Various Values of n and λ for an Elastically Restrained Circular Plate.*

λ	$n = 0$		$n = 1$	$n = 2$
$\to 0$	3.20	6.30	4.61	5.90
10^{-3}	3.19	6.30	4.61	5.90
10^{-2}	3.17	6.25	4.57	5.85
10^{-1}	2.96	5.93	4.31	5.55
10^{0}	2.46	5.55	3.87	5.17
$\to \infty$	2.22	5.45	3.73	5.06

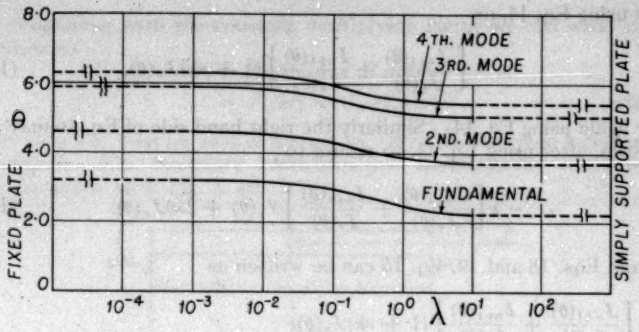

FIG. 4. Variation of θ with elastic λ restraint for a circular plate.

values of clamped plate corresponding to $\lambda \to 0$ have been taken from Prescott (10).

CONCLUSIONS

The curves in Figs. 2 and 4 enable us to determine the elastic restraint for a circular plate by measuring the deflection at the center in the one case and the frequencies in the other case. Both in bending and vibration problems it is seen that the values for the two limiting cases of simply supported edge and clamped edge are reached asymptotically.

Acknowledgments

The author's thanks are due to Mr. C. V. Joga Rao for suggesting the problem and for his guidance. The author is grateful to the Government of India for the award of a Research scholarship during the period of investigation.

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Electronics Probes Human Heart.—It is now possible to take blood samples from within a beating heart and simultaneously make extremely accurate measurements of in-heart blood pressure. A tiny pressure pick-up device threaded through arteries directly into the heart will help diagnose various heart defects. The device was described by Alan Warnick, supervisor of the electronics department, of the scientific laboratory of Ford Motor Company's engineering staff, at the recent IRE convention. The instrument, thinner than a match stick and only half an inch long, was developed in cooperation with the Henry Ford Hospital.

The system measures blood pressure actually inside the heart, and because of this is free from interference and distortions. Blood pressure is converted into electrical signals so small that the electronics laboratory had to develop a new amplifier to bring the electrical pulses up to a level that could be recorded.

Satisfactory results were obtained in animal experimentation and the instrument now has been used on more than a dozen adult patients.

The type of diagnostic research being done is called "cardiac catheterization," and involves studies of blood pressure and pulse in the heart, plus a sampling of blood taken from one of the chambers.

The sensing element employs the strain-gage principle. Within the device a miniature bellows is activated by cardiac pressure. The bellows presses against a coil of wire that encircles it, wire many times thinner than a human hair. This wire has a current of electricity running through it and when stretched by the bellows, it increases its electrical resistance.

This electrical change, amplified and recorded, has a higher natural frequency than other types, above 2000 cycles per second. Because of this, its readings are particularly valuable for work with infants and children where faster heartbeats increase the problem of interference by vibration.

Non-Depressant Anesthetic.—Results of a preliminary study involving use of Sernyl, a new, non-depressant, rapid-acting anesthetic, were delivered recently before the International Congress of Research Anesthetists.

Dr. F. E. Greifenstein, Professor of Anesthesiology at Wayne State University's College of Medicine in Detroit, reported that the injectable anesthetic opened avenues to "additional work with non-barbiturates as anesthetics." The doctor and his associates, Dr. Marion DeVault, Dr. J. Yoshitake and Dr. John E. Gajewski, observed the new anesthetic in 70 patients at Detroit's Receiving Hospital.

Sernyl does not depress respiration or circulation and induces a temporary amnesia in patients, Dr. Greifenstein said. As a result, they are unable to recall any of the operating room procedures other than the initial injection of the drug.

He said Sernyl leaves patients insensible to pain, is adequate for superficial surgery, and, in combination with relaxing drugs, can be used satisfactorily in major surgery.

Sernyl was developed by Parke, Davis & Company, world-wide pharmaceutical firm, and is the company's registered trademark for 1-(1-phenylcyclohexyl) piperidine monohydrochloride.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING
May 21, 1958

The Stated Meeting of The Franklin Institute was held at 8:15 P.M. in the Lecture Hall. Mr. Wynn Laurence LePage, President, called the meeting to order with approximately 270 members and guests in attendance.

Mr. LePage reported that the minutes of the Stated Meeting of March 19, 1958 were printed in the April issue of the JOURNAL and the minutes of the Special Meeting of April 2, 1958 and the Stated Meeting of April 16, 1958 were printed in the May issue of the JOURNAL. There being no corrections or additions these minutes were approved as published.

Mr. LePage announced that the Board of Managers, at its meeting this afternoon, had nominated Mr. Donald Wills Douglas, of Santa Monica, California, Franklin Medalist for 1958, to Honorary Membership in The Franklin Institute, and, in accordance with our By-Laws, his name was presented to the membership for election. On motion duly made and seconded, the following Resolution was presented and unanimously adopted:

> RESOLVED, That Donald Wills Douglas, of Santa Monica, California, Franklin Medalist for 1958, be elected to Honorary Membership in The Franklin Institute.

There being no further business, the President introduced the speaker of the evening, Mr. Ferdinand D. Bickel of the Explosives Division of E. I. duPont de Nemours & Company. Mr. Bickel is considered the top "blasting" expert in the world, and has been involved in blasting projects throughout the world. Mr. Bickel gave a most interesting talk on the demolition of Ripple Rock, which is in Seymour Narrows, one hundred miles north of Vancouver on the west coast of Canada. Many vessels have been damaged and lives lost due to collisions with the twin peaks of Ripple Rock which came within nine feet of the surface in the much traveled inside passage to Alaska. Mr. Bickel's talk clearly pointed up the research and engineering work necessary to accomplish this important project. His talk was well illustrated with slides, motion pictures and charts and both the dramatic aspects and uniquely challenging technical features of the Ripple Rock operation were fully appreciated by his audience.

After enthusiastic applause Mr. LePage opened the question period which brought forth many thought-provoking questions.

The meeting was adjourned by the President at 10:15 P.M.

WILLIAM F. JACKSON, JR.
Secretary

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, May 14, 1958.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MAY 14, 1958.

MR. JOSEPH GRAY JACKSON in the Chair.

The following reports were presented for final action:

No. 3189: Work of J. C. Patrick on Thiokol.

This report recommended the award of an Elliott Cresson Medal to Joseph C. Patrick, of Yardley, Pennsylvania, "In consideration of his discoveries in the field of polysulfide polymers, and of the new processes devised by him involving the combination of chemical compounds for the manufacture of a synthetic rubber having special properties adapted to critical uses and to the manufacture of a solid rocket fuel."

No. 3341: Clark Medal.

This report recommended the award of the Walton Clark Medal to Frederic O. Hess, of Philadelphia, Pennsylvania, "In consideration of his many contributions to the gas industry, in the improvements and developments of new processes for the use of gas industrially."

No. 3342: Ballantine Medal.

This report recommended the award of the Stuart Ballantine Medal to Harald Trap Friis, of Rumson, New Jersey, "In consideration of his many important contributions to the science of radio-communications during a lifetime of consistently productive research in this field, through which this science has been so notably advanced."

D. S. FAHRNEY,
Secretary to Committee

LIBRARY

The Committee on Library desires to add to the collections any technical works that members may wish to contribute. Contributions will be gratefully acknowledged and placed in the Library. Duplicates received will be transferred to other libraries as gifts of the donor. 438 items have been added the past month.

Photostat service. Photostat prints of any material in the collections can be supplied on request.

The Library and reading room are open from 9 A.M. until 5 P.M. on Mondays, Tuesdays, Thursdays, and Fridays; 2 P.M. until 10 P.M. on Wednesdays; and from 9 A.M. until Noon on Saturdays.

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MEMBERSHIP

The following elections were approved on May 1, 1958 by the Membership Committee by authority given to it by the Board of Managers.

ANNUAL

Lorenzo J. Adams
V. Frank Arpajian
Ronald Baird
W. S. Bradfield
Carlos F. Chong
Mario A. Conetta
Jerome Corwin
William T. Crawford
James R. Custer
David Cutler
John J. Douglass
H. Duke Edwards
Eugene B. Eipper
Sidney Elkman
Harry Estersohn
Harold G. Etchen
Palmer D. Evans
Leonard Eveler
Ernest J. Ewald
Charles A. Fahy
Marvin S. Feller
James E. Foley
Raymond G. Frank
William E. Gildersleeve
Eugene L. Green
Herbert T. Greene
H. Thomas Hallowell, Jr.
H. D. Honan, Jr.
Edward J. Kessler
Ezra S. Krendel
Amor L. Lane
Frederick M. Levin
Sydney A. Martin
J. V. Miller, Jr.
Leslie B. Moxon
Alexander Nagy, Jr.
Martin Pearlstein
Mrs. Zdenka Placek
Peter Taman
Henry J. Repsher
William H. Schwemlein
Terry Shaner
Joseph F. Sibley
Charles L. Starbuck
George J. Steinbronn
John W. Streeter
Richard V. D. Strong
Peter S. Zeimann

EDUCATOR

David Hudnut
Elaine Kapust
Chester W. Lathrop, Jr.
Adrian M. McDonough
LeRoy B. Scott
James E. Seitz

NON-RESIDENT

Arnold S. Blum
Malcolm N. Brown

NECROLOGY

Frank Beemer '48
H. B. Frazer '43
Chauncey Harllee '48
D. H. Redmond '43
Rowland E. Roberts '36
C. H. Speckman '42
Walter C. Wagner '42

New Color Projection Technique.—A colorless, transparent slide produced a full-color photograph when projected on a screen during a demonstration at a recent meeting of the Optical Society of America.

Dr. William Glenn, a physicist at the General Electric Research Laboratory, Schenectady, N. Y., presented a technical paper describing his new technique for recording "color information" in tiny ripples on a clear gelatin film.

The General Electric scientist said the new color projection technique may have eventual importance in color television for both home and theater applications. His paper described how his method might be adapted to projection TV systems, making possible simultaneous projection of primary colors in perfect register on the screen.

The ripples in the film serve as a diffraction-grating system, and a special projector is used to "translate" the diffraction patterns—overlapping series of light waves—so that brilliant primary colors are projected on the screen at the same time. The slides demonstrated by Dr. Glenn were in the standard 2 × 2 size, and because the diffraction ripples were only a thousandth of an inch apart, the projected picture had a resolution—sharpness—comparable to a good TV picture.

Dr. Glenn also showed how the color information could be recorded on black-and-white film, and his audience of physicists had the unique experience of seeing an ordinary-appearing black-and-white slide produce a full-color picture on the screen. He said that by placing a special mask in front of the film in an ordinary camera, color photographs can be taken directly on black-and-white negative materials. (The diffraction-grating system produces a "positive" color picture if the monochrome slide is either positive or negative.) Dr. Glenn emphasized that the technique has not yet been developed for wide use in photographs.

"We are much more interested in applying the idea to television," he said.

An unusual aspect of Dr. Glenn's system is that it makes possible a full-color picture produced by two primary colors, one fixed and one variable, instead of the usual system of three fixed primaries. He showed today how the "one-fixed-and-one-variable" method actually reproduces a greater part of the visible spectrum than the ordinary three-primary system.

For his clear and transparent "color slides," Dr. Glenn prints the diffraction-grating ripples on chromated gelatin. His black-and-white "color slides" utilize fine-grain photographic film. Dr. Glenn suggested that the Swiss Eidophor system for black-and-white TV projection, in which diffraction gratings are written onto an oil film by an electron beam, might be modified for simultaneous color projection using the new technique.

Possible advantages of the new system in color television would include high brightness, perfect color register, and greater simplicity than color-TV systems now in use. General Electric officials emphasized, however, that Dr. Glenn's presentation was "a technical paper describing a laboratory achievement and definitely not a product announcement."

THE FRANKLIN INSTITUTE MUSEUM
A. C. CARLTON, DIRECTOR

RESEARCH AND PATENTS

BY

THOMAS COULSON

The United States Patent Office, established to "promote the Progress of Science and the useful Arts," is now one hundred and sixty-five years old and is, therefore, a venerable institution. Since its beginning, the U. S. Patent Office has been the keystone in the structure of American industry and business. The grant of a patent, by awarding exclusive rights for a limited period to the fruits of the labors of imaginative thinking, gave encouragement and incentive to undergo the dogged, costly labor, the discouragements and disappointments that attend creative effort.

When the patent laws were formulated the men who wrote them were thinking only of the individual inventor, and no blame can be attributed to them if they failed to foresee the time would come when inventions and discoveries would be the products of team work. The great research organizations of large companies, employing hundreds of research workers spending millions of dollars annually, could not be visualized by the most unfettered mind of those days. Yet the system has worked well. The Patent Office still recognizes only the individual, whether he works alone in a basement workshop or whether he is part of a large group working in a well equipped laboratory. This is as it should be. For the fact is that in any invention the initiative comes from an individual, and to him should be awarded the prestige and prominence of priority.

A patent is an inventor's best protection. It is not a guarantee of profit. In order to obtain the protection the inventor must apply for a patent, giving a full description of his invention and a statement of claims setting out what features of the invention are original. The inventor receives a report upon his application, affording him an opportunity to reword or amend his claims. Finally, the inventor will be told which of his claims will be allowed. With this information in his possession, the inventor must decide for himself whether the admitted claims are important enough to make a full disclosure that will be required if a patent is granted. He must weigh the value of the prestige that will be his and evaluate the income he may justifiably receive from the seventeen years life of the patent.

We must not suppose the nature of his patent will remain unknown to others. On the contrary it will be submitted to minute examination. The patent disclosure will be published by the Patent Office and printed copies can be consulted in many libraries. The Franklin Institute

Library has an extensive file of American patents, and inventors would be well rewarded if they studied them before applying for their patents. Such precautions save much time, labor, and money in sparing a research worker from wasting effort upon a problem that is already protected by patent.

It might be supposed the patent system would stifle competition. Such a supposition would be erroneous. The system is a potent stimulus to competition. Another feature of the patent system which its framers could not have foreseen is that patent protection permits the circulation of information throughout the industry. Not only does this save wasted effort by workers in the same field (provided they are willing to deny themselves time from laboratory experiment to devote it to literary research), but this diffusion of knowledge opens up fresh vistas unperceived by the inventor. The utility of the patent may be apparent to a competitor seeking to reduce his manufacturing costs or to enlarge his operations, and this may induce him to adopt the patent under license, to the further enrichment of the patent holder.

During the months of March and April, the Museum was fortunate enough to secure for display fifteen excellent exhibits, illustrating the value of research and the patent system to the chemical industry. These exhibits furnished an eloquent tribute of an industry to research and its protection by patent. The representative companies embraced the wide field of home, farm, industry, and national security in the variety of their products. Visitors were privileged to see how research, adequately protected from piracy by the patent system, is encouraged to produce newer and better ways of doing things, newer and better products for improved health, better living, greater comfort and security for all. Several of the exhibits bore testimony to the effectiveness of the patent system in promoting the growth of a small concern into a vast organization under the shelter of the protection it affords.

THE FRANKLIN INSTITUTE LABORATORIES
NICOL H. SMITH, DIRECTOR

THE STUDY OF EXPLOSIONS BY FLASH ABSORPTION SPECTROSCOPY

BY

FRANCIS R. TAYLOR[1]

Short-lived molecular species occurring during gas explosions have been detected by means of flash ultraviolet absorption spectroscopy. G. Porter (1), originator of this technique, collaborated with R. G. Norrish (2) in using this method to study hydrogen-oxygen explosions. Their method consisted of initiating the reaction by means of a photolysis flash, and then taking the absorption spectrum of the reacting gases by a second flash which occurred a few milliseconds later. The triggering of the flash was done by electrical contacts on the surface of a rapidly rotating disk. This disk also acted as a shutter, excluding from the spectrograph the light from the photolysis flash lamp, but passing the emission from the flash lamp supplying the continuous background for the absorption spectrum.

At The Franklin Institute Laboratories we have adapted this flash technique to the study of gaseous explosions which are not induced by photolysis but which occur spontaneously as a result of the thermal energy present at the temperature of mixing (3). This method has allowed us to follow the increase and decrease in the concentration of unstable radicals and molecules during explosions of carbon disulphide and oxygen (4), and propane and nitrogen dioxide (5) which last only a few milliseconds. Our apparatus is much smaller and simpler than that described by Porter.

Our present technique consists of rapidly introducing mixtures of reactants into 4 cm. diameter quartz absorption tubes 20 or 55 cm. long which are maintained at a constant temperature in a large furnace. Light emitted by the reacting gases is transmitted through the heavy walls of the furnace by a quartz rod, acting as wave guide, to a sensitive photomultiplier tube. Part of the output of this detector is utilized to actuate a thyratron delay circuit—firing a high intensity flash tube at any predetermined time during or after the reaction. Part of the output is also transmitted to an oscilloscope, where the resulting trace of the total radiation from the reacting gases and the flash tube are photographed. Finally the light from the flash tube passes through the reacting gases and into a large Bausch and Lomb spectrograph, where an absorption spectrogram is produced. The flash tube used in this work consists of a 4 mm. i.d. quartz tube having two internal tungsten electrodes and one external triggering electrode located midway between

[1] Research Chemist, Chemical Kinetics and Spectroscopy Section.

them. The flash tube contains 10 to 12 cm. Hg of xenon and is energized by a photoflash 25-microfarad capacitor charged to 5000 volts. Oscilloscope traces of the light emission from this source shows it to have a discharge time of approximately one millisecond.

The technique of flash ultraviolet absorption spectroscopy can be applied to nearly any rapid gas phase reaction where chemical species are produced which have absorption bands in the visible or ultraviolet.

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(3) Sponsored by Wright Air Development Center, Dayton, Ohio.
(4) A. L. MYERSON, F. R. TAYLOR AND P. L. HANST, *J. Chem. Phys.*, Vol. 26, No. 5, 1309–1320 (1957).
(5) F. R. TAYLOR, B. FAUNCE *et al.*, ASTIA Document No. AD118105, WADC Technical Report 57-138, June 1957.

***Seminar on* Charge and Specific Ion Effects on Sedimentation in the Ultracentrifuge.**—DR. KAI O. PEDERSEN of the Institute of Physical Chemistry, University of Uppsala, Uppsala, Sweden, addressed the seminar of the Biochemical Research Foundation on Wednesday afternoon, January eighth, 1958.

Up to the present time very little attention has been paid to the influence that electric charge may have on the sedimentation of macro-ions. Except for an article by Tiselius (1) in 1932 and a preliminary study by the author (2) at the same time, no systematic investigation seems to have been made. A few papers dealing with special cases may be mentioned (3–5).

Generally in praxis a sufficient quantity of neutral salt is added to the solution to be centrifuged so that any primary charge effect is swamped. Furthermore, the sedimentation experiments are frequently carried out in the neighborhood of the isoelectric point, where such effects are less pronounced. In recent years, however, interest has developed in the effects produced on macro-ions such as proteins when they are removed from their isoelectric points and when they pass the limit of their stability range. In such cases very marked charge and salt effects may occur, and if disregarded they may lead to erroneous interpretation of the experimental results. In order to study these problems a systematic investigation of charge effects was begun three years ago at the Institute of Physical Chemistry at the University of Uppsala, Sweden. This study is still in progress, but it was considered timely to give a first preliminary report of some of the results obtained so far.

Let us first consider an example. Suppose we have an isoelectric electrolyte-free protein solution and to this add a small amount of HCl so as to bring the pH of the solution somewhat below the isoelectric point for that protein. We have now a protein chloride solution. If this is centrifuged, the different sedimentation tendencies of the protein ion and its counter-ion (Cl^-) result in a microscopic separation of charge both at the bottom of the cell and in the sedimenting boundary in accord with Poisson's relation

$$d^2\psi/dx^2 = -4\pi\rho/\epsilon,$$

where ψ is the potential at a point where the charge density is ρ, and ϵ is the dielectric constant of the medium containing the charges. This equal and opposite separation of charge establishes in the intervening column of solution an electric field, $d\psi/dx$, which, in conjunction with

the centrifugal field, causes the protein and its counter-ion (Gegenion) to sediment with the same velocity, slower for the protein ion and faster for the Cl^- ion. With increasing charge on the protein, $d\psi/dx$ increases and the sedimentation velocity for the protein decreases. This we call the *primary charge effect.* It is most pronounced in solutions of low ionic strength in which the concentration of the protein salt is large in comparison with that of the other ions present.

A *primary salt effect* is produced whenever neutral salts (or excess acid or base) are added to a solution showing a primary charge effect. By the addition of NaCl, for example, to the protein chloride solution, the electric field is decreased, since the Cl^- ions available to balance the charge on the protein molecule are now present at a higher concentration than the protein ions and thus do not have to move as rapidly as the protein ions to preserve electrical neutrality. The increase in s_{20} is most pronounced upon the first addition of salt and becomes progressively smaller as s_{20} asymptotically approaches a value close to that for the isoelectric protein.

With increasing concentration of neutral salt, the ratio of protein concentration to total salt concentration approaches zero, and the electric field decreases; but it does not approach zero unless the ions of the neutral salt have identical sedimentation constants. If the ions have sedimentation coefficients which are different, then the limiting value ($c_0 = 0$) of s_{20} for the protein depends upon the nature of the salt ions. This may be called the *secondary salt effect.* It occurs in every solution containing macro-ions where

$$\sum_1^n m_i c_i s_i \neq 0.$$

In this expression m_i, c_i and s_i are the valencies (positive for cations, negative for anions), molar concentrations, and sedimentation coefficients, respectively, for the macro-ion ($i = 0$) and all the other ions present in the solution ($i = 1,2,3, \ldots n$). Like the primary charge effect, it is due to the different sedimentation rates of the various ions in the solution, particularly to the difference in the s values for the small ions of the added salt. The secondary salt effect is often most pronounced in the more concentrated salt solutions.

In addition to the effects described above and caused by what we may perhaps call physical factors, there are other effects of a more chemical nature that may complicate the experimental results. Thus in many cases a specific binding of certain ions (especially anions) to the macromolecules obtains, which produces a change in the net charge on the macro-ion (at constant pH) and increases the value of $M(1 - \bar{v}\rho)$. Both these results of ion binding to the macromolecule may give rise to changes in s.

For the interpretation of results obtained from sedimentation experiments with solutions containing charged macromolecules it is important to know something about the magnitude of these effects. We are also interested in knowing under which conditions they are minimal, and what value s would have in solutions of the same pH and ionic strength if secondary salt effects could be eliminated. This can be done when the sedimentation constants and equivalent conductivities of the individual ions in the solution are known.

It can be shown (2) that the *observed* sedimentation coefficient for the macro-ion, s_0', is related to the "true" s_0 in the following way:

$$s_0' = s_0 - \frac{u_0}{\omega^2 x} \cdot \frac{d\psi}{dx} = s_0 - u_0 \frac{\sum_0^n m_i c_i s_i}{\sum_0^n m_i c_i u_i},$$

where u_i is the electrophoretic mobility of the i-ion ($i = 0$ for the macro-ion) in cm. per sec. for a potential gradient of 1 e.s.u. per cm. From this equation it is seen that $s_0' = s_0$ either when $u_0 = 0$ or when $d\psi/dx = 0$. The first case, $u_0 = 0$, is restricted to solutions in which the macromolecule is isoelectric, and this condition may occur at different pH values depending on the ionic strength of the solution and the nature of the anions present. The value of $\sum m_i c_i u_i$, which is proportional to κ, the specific conductivity of the solution, is always positive. Therefore, if $d\psi/dx$ is equal to zero, $\sum m_i c_i s_i$ must also equal zero. Whether or not secondary salt effects will occur should consequently be readily determined if m_i, c_i and s_i are known for all the ions in the solution to be investigated. A preferable approach, however, has been a study of the macro-ion in solutions of constant pH and ionic strength, containing the same anions throughout, but with the cation systematically varied to give both positive and negative values of $d\psi/dx$. Each of these systems has been run at various protein concentrations to determine the limiting sedimentation constant, s_0', corresponding to $c_0 = 0$. On the assumption, as a first approximation, that u_0 is independent of the nature of the cation, the resulting values of s_0' have then been plotted against the corresponding calculated values of $\frac{k}{\omega^2 x} \cdot \frac{d\psi}{dx}$ to obtain $s_0' = s_0$, the value corresponding to $c_0 = 0$ and $\frac{k}{\omega^2 x} \cdot \frac{d\psi}{dx} = 0$.

The sedimentation constants for the various ions may be calculated from two kinds of experiments, viz.: (1) self-diffusion measurements, and (2) combined conductivity and transport number determinations. In addition to such measurements, however, the partial molar volume,

$\bar{V}$, for the ion must be known. Such values for $\bar{V}$ are to be found in the literature for a great number of ions (6). They have been calculated from different assumptions. In this case it has been assumed that $\bar{V}_0 = 0$ for the hydrogen ion.

The limiting sedimentation coefficients for a number of ions have been calculated for water at 25° C. For the cations, s_{25} varies from 0.001 S* for NH_4^+ to 1.52 S for the Tl^+ ion. For the anions, the values increase from 0.08 S for the acetate ion to 0.75 S for the I^- ion. For salts, values varying from $s_{25} = 0.07$ S for LiCl to $s_{25} = 0.84$ S for CsI were computed.

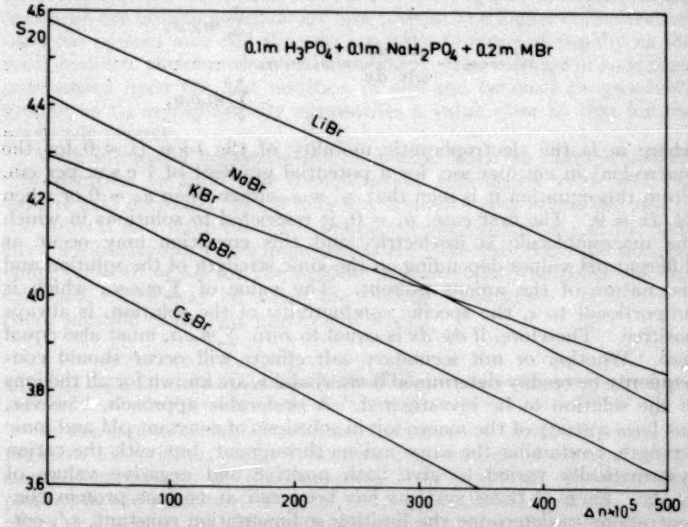

FIG. 1. Variation of s_{20} for BSA with protein concentration in solutions of various 0.20 M alkali bromides in an acid phosphate buffer solution of pH ~ 2.

Examples have also been compiled showing the values of $d\psi/dx$ for a number of neutral salts and buffer salts. These tables show that very small secondary salt effects should be present in solutions of the following salts: NaF, NaCl, KNO_3, K_2HPO_4 and $MgCl_2$.

Throughout the investigation described here bovine serum albumin (BSA) was used as model substance because it has a suitable s_{20}, a broad stability range, and may become highly charged. For the purpose of this investigation it seems unimportant whether in some of

* 1 S (Svedberg) = 1×10^{-13} c.g.s. unit.

the acid solutions the BSA is in the native or in the denatured form as long as a possible denaturation is independent of the nature of the cation.

The experiments described in the following were all carried out at a pH either around 9 or around 2, where one peak only is observed in the sedimentation diagram. In the region between pH ~ 2 and pH ~ 4 the diagrams are often asymmetrical and may show two peaks, depending upon the pH and the anions present in the solution (for references see 4, 5).

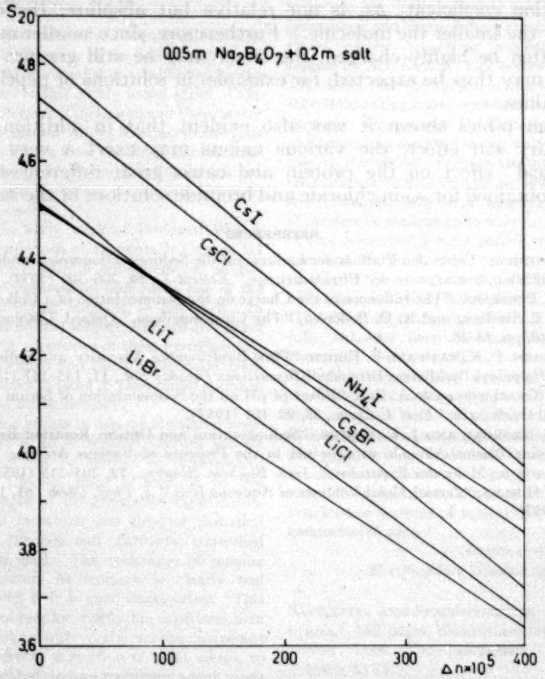

FIG. 2. Variation of s_{20} for BSA with protein concentration in solutions of various 0.20 *M* alkali salts in 0.05 *M* $Na_2B_4O_7$. pH ~ 9.

As examples of the variation of the sedimentation coefficient with protein concentration and the nature of the salt medium, two of the diagrams shown are given here (Figs. 1 and 2). In the first diagram the variation of s_{20} with protein concentration is shown for the various alkali bromides in an acid phosphate buffer solution of pH ~ 2. The

sedimentation coefficient decreases when the cation increases in mass from Li^+ to Cs^+. The opposite is the case for the alkaline borate buffer of pH $\sim$ 9, where the sedimentation is faster in CsBr than in LiBr.

From Figs. 1 and 2 it is clearly evident that the nature of the cation is quite important for the results that may be obtained in solutions of charged macromolecules. In the case of serum albumin the difference between the values obtained in solutions containing Li^+ or Cs^+ may amount to about 10 per cent. As, however, the variation in the sedimentation coefficient, Δs, is not relative but absolute, the effect is greater the smaller the molecule. Furthermore, since smaller molecules may often be highly charged, the effect may be still greater. Large effects may thus be expected, for example, in solutions of peptides and protamines.

From tables shown it was also evident that in addition to the secondary salt effect, the various anions may exert a very specific "chemical" effect on the protein and cause great differences in the values obtained for s_{20} in chloride and bromide solutions of the same pH.

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BOOK REVIEWS

PROGRESS IN SEMICONDUCTORS, VOLUME II, edited by Alan F. Gibson, P. Aigrain and R. E. Burgess. 280 pages, illustrations, 6 × 9¼ in. New York, John Wiley & Sons, Inc., 1957. Price, $10.50.

The second volume of *Progress in Semiconductors* fully lives up to the high standards of interest and workmanship established by the first volume in this highly useful series. The paper by Herman, Glicksman and Paramenter on Semiconductor Alloys lives up to the high standard of work from the RCA Laboratories in Princeton. Much information is still required, however, before a complete interpretation is possible.

Since the early work of Professor Welker on the compounds of elements in Groups III and V, this branch of semiconductor physics has grown enormously and forms one of the most challenging subjects in the field. In their paper on this subject, Cunnell and Saker describe the properties of these materials in a straight-forward and easily understood manner and indicate some of their applications in semiconductor technology.

Radiation effects in reactor irradiated semiconductors is an extremely complex subject and should not be studied as fundamental physics. Clearly, what is required in this work is more of the controlled type experimentation in which one type of radiation alone, of known and narrowly controlled energy, is used. The technique of putting semiconductors in reactors is clearly bad science even if it is good engineering. This criticism of reactor irradiation experiments in no way reflects unfavorably on the competent and interesting exposition of what seems, to the uninitiated, to be a technique which raises more fundamental problems than it can solve. On the other hand experiments with accurately controlled radiation can make valuable contributions to both semiconductor theory and the understanding of the effects of reactor irradiations.

The case for horizontal crystal growing is made out admirably by Cressell and Powell who have obtained low dislocation densities by this method. Crawford Dunlap provides an excellent review of a field which still deserves more detailed work, namely, impurities in germanium. High electric field effects in germanium and silicon, where much is known about carrier scattering mechanisms, is treated by Mr. Gunn. This review is timely and shows that the subject is capable of further illuminating transport effects in semiconductors. The paper of Dr. Curie on Theories of Electro-luminescence is also welcome addition to the series. Perhaps the outstanding contribution to this volume is, however, the theoretical article by Dr. Rose on the life time of carriers in semiconductors.

This collection of eight papers provides an excellent demonstration of the continued vitality of the semiconductor field as a branch of fundamental solid state physics. Volume II of *Progress in Semiconductors* is a beautifully balanced blend of experimental and theoretical research. The reviewer hopes, however, that attention will be given to the newer fields of surface physics and magnetic spin resonance in the next volume. There is little doubt that, in the capable and discriminating hands of general editor Dr. Gibson and his associates, this series is going to continue to fulfill a growing need for good introductory articles and surveys of topical fields in semiconductor physics.

NICHOLAS FUSCHILLO
The Franklin Institute Laboratories

SATELLITES AND SPACEFLIGHT, by Eric Burgess. 159 pages, illustrations, 5½ × 8½ in. New York, The Macmillan Co., 1958. Price, $3.95.

The author of this book is one of the early and ranking leaders in the space travel field and as such has exercised a profound influence on the thinking of those who are venturing in the field.

While his other books were written as a professional, his latest work is "pitched to the level of the layman." Essentially *Satellites*

and Spaceflight is a descriptive narrative in which the layman can visualize the research, training, preparation and disciplines which enter into the launching of a satellite. Once satellites become commonplace then the next step—that of man going into space—is covered.

The author uses a clever arrangement in the writing of this book. The first of the six chapters is devoted to some astronautical background and to a slight historical exposition. Then the details of the Vanguard are covered. This reviewer would love to see the Russian satellite launching vehicles covered as well, but this is wishful thinking. The author has canvassed the field as to the uses of satellites—both instrumented and non-instrumented—and has shown how much intelligence can be derived from these satellites.

His second section deals primarily with the human body in space. This, to the reviewer, is one of the most fascinating aspects of the space travel problem. Here again is a skillful digest of our present day knowledge and speculations concerning our ability to live in a completely alien and unfriendly environment. The utilitarian aspects of the space station conclude this chapter.

Probing into space is next covered with the logical stepping off point—the moon—explored. The author speaks of the various types of moon-exploration rockets but does not cover the entire field. Travel beyond the moon—to Mars—is then portrayed with a rather complete picture presented. This analysis was made some years ago and was reported in the *Journal of the British Interplanetary Society*. Here refinements are presented which picture the Martian problem in a clearer light.

Eric Burgess has succeeded in writing a completely fascinating book which adds to the background of space travel. *Satellites and Spaceflight* is one of the author's best works and merits the confidence of his readers.

I. M. LEVITT
Fels Planetarium of The Franklin Institute

QUEUES, INVENTORIES AND MAINTENANCE, by Philip M. Morse. 202 pages, diagrams, 6 × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $6.50.

This publication is the debut of the Operations Research Society of America into the field of monography. The text fills a need for some of the more fundamental queuing theories to be neatly gathered together for ready reference. The treatment given the subject is naturally more thorough than encountered in a few chapters of a text on probability theory.

The author has followed a clear line of development starting with the simple case of a single channel operation with exponential arrival and service time distributions. He finishes with multi-channel operation with complex arrival and service times being simulated by the method of Erlang. Parallel with the development of the theory, consideration is given to the estimation of "best" operating combinations as dictated by economics. The areas of application are evident from the title.

If one has been exposed in some degree to the formulation and solution of queuing problems, the book is ideal. However, it is not recommended as a text, per se, for the uninitiated. Certainly to fill this need, the initial section of the book, which covers probability models, needs enlargement. It is complete but bleak.

At the same time, the student, before too long, may succumb to the false notion that the use of complex mathematical simulation of distribution is the only available tool toward problem solution. To round out the presentation, some space should have been allotted to the use of Monte Carlo techniques, and an idea given where their application is more feasible than mathematical models.

ALOYSIUS J. POLANECZKY
The Franklin Institute Laboratories

TRANSISTOR PHYSICS AND CIRCUITS, by Robert L. Riddle and Marlin P. Ristenbatt. 428 pages, diagrams, 6 × 9 in. Englewood Cliffs, N. J., Prentice-Hall, Inc., 1958. Price, $10.00.

Just ten years have passed since the transistor first made its appearance. In this time, transistor technology has developed with extreme rapidity, evidenced today by the number of consumer products boasting of transistor circuitry. Steadily declining costs and reproducibility of characteristics now permit the transistor to compete successfully with the vacuum tube in a growing number of applications. Those trained in vaccum tube

techniques are finding it necessary to re-orient their thinking, sometimes to a substantial degree, in order to be able to apply these new devices with facility. With the future of the transistor well established, it has become essential for all electrical engineers and designers to acquire this reorientation and necessary background in the shortest possible time.

Messrs. Riddle and Ristenbatt have attempted to ease the problem of introducing the technician and designer to the techniques of transistor theory and circuit design by the introduction of the present text. *Transistor Physics and Circuits* assumes no pre-requisites other than a basic high school knowledge of physics and algebra. Thus the level coincides directly with the Technical Institute level. The material of the book is divided into two parts, covering first the physics and then the circuit aspects of transistors. The section on physics, extending through the first five chapters, covers such fundamental subjects as crystal structure and properties, energy levels, effects of impurities, physical action of transitors, and a summary of various transistor types. Chapters 6 through 15 are devoted to an analysis of various circuit aspects. Here performance calculations and some design procedures are studied for both small-signal and large-signal transistor circuits. The topics of frequency response and feedback methods are treated thoroughly. Problems are provided at the end of each chapter give the reader opportunities for applying the theory covered in that chapter. An appendix contains a set of selected transistor characteristics as well as a short treatment of determinates and the pertinent relations between the various matrices used with transistors.

In this reviewers opinion, the authors have admirably achieved their goal of presenting a concise, lucid introduction to transistor circuit design theory with a minimum of complication. One disappointing feature is the lack of any bibliography. While the text is designed to be self explanatory, the inclusion of a bibliography would enhance the value of the book, permitting more advanced students to easily find more details on subjects of particular interest. The real value of the book lies in its very clear and practical coverage of an important and relatively complex subject. Most of the texts published heretofore, require much more background of the reader. This book, if digested first, would certainly improve the appreciation of more advanced treatments. In itself, *Transistor Physics and Circuits* presents adequate background for the efficient handling of most ordinary transistor circuit applications, and should be received enthusiastically by electrical engineers and designers seeking to acquire a practical working knowledge in this field.

D. L. BIRX
The Franklin Institute Laboratories

THERMODYNAMICS OF ONE-COMPONENT SYSTEMS, by William N. Lacey and Bruce H. Sage. 376 pages, 6 × 9 in. New York, Academic Press Inc., 1957. Price, $8.00.

This book is even more restricted than its title would indicate. The viewpoint is that of the engineer, with English units used throughout, and a limited number of topics is treated.

The reader is apparently assumed to have some knowledge of thermodynamics. Although the first part includes the basic principles, it is more in the form of a review, and the explanations are inadequate for an introduction to the subject. This section includes units, partial differential equations, general concepts, the first law and its consequences, the second law, a tabulation of thermodynamic functions, and cycles. The equations are completely derived. Then various basic systems and processes are covered: homogeneous gases and liquids, heterogeneous systems, irreversible processes.

Flow processes are discussed at length. The general principles are derived and applied to several cases, including flow meters, nozzles, and jet pumps. Other subjects are: various types of cycles, turbines, reciprocating engines, refrigeration, and low temperature liquefaction of gases. Numerous examples are worked out at intervals, and a comprehensive list of references is given at the end of each chapter. Included in the appendix are derivations of the more difficult equations and a series of problems.

This volume would serve as a useful introduction to the various applications for an engineering student already familiar with the fundamentals of thermodynamics.

NEWTON MCCREADY
Philadelphia Quartz Co.

BASIC FEEDBACK CONTROL SYSTEM DESIGN, by C. J. Savant, Jr. 418 pages, 6 × 9 in., drawings and photographs. New York, McGraw-Hill Book Co., 1958. Price, $9.50.

Although one might be inclined to observe merely that another book on servomechanisms has found its way to the shelves, Dr. Savant's new book is an exceptionally good and useful one from the standpoint of the general electrical engineer. The feeling received upon examining its contents is that here is a real guide to practice which can be put immediately to use. It is certainly not cluttered with academic, mathematical exercises of ridiculous complexity.

Such comments are definitely not to imply that Dr. Savant's book is elementary and lacking rigor. On the contrary, he has done two things of courage; first, he has broken with tradition which presents only frequency analysis techniques. By introducing root-locus methods, which are usually exiled to the connoisseur's corner of servomechanism design, he has shown leadership. The root-locus pictures the character of the servo system and fits more closely the trial and error method by which control devices are developed. The method is not extremely difficult and can surely be taught at the undergraduate level.

Secondly, Dr. Savant has added numerous examples of practical design without fear of humbling his treatment. The inclusion of numerical illustrations and reference to commercial devices has not detracted from the technical qualities of text. On the contrary, this author has selected carefully the basic elements of feedback control design including the initial attainment of the system differential equation, its solution, methods of equalization, and a brief treatment of nonlinearities.

Undoubtedly the contacts which Dr. Savant has had with industrial problems are partly responsible for his clear conception of this book's contents. It is a problem to know what to include in roughly 400 pages that will do justice to such a large and dynamic subject as control system design.

This book could easily achieve its goal of giving the student substantial facility with servo design if a laboratory course could be taken in conjunction. Of course, the book is not aimed at the graduate level. It will, however, be of use to practicing engineers who are called upon to use servomechanisms in solving various problems. The servo specialist will probably not find the book to be more than a convenient summary.

C. W. HARGENS
The Franklin Institute Laboratories

THE PHYSICO-CHEMICAL EFFECTS OF PRESSURE, by S. D. Hamann. 246p ages, 5½ × 8½ in. New York, Academic Press, Inc., 1957. Price, $8.50.

In studying the variables which influence the physico-chemical behavior of materials, man has tried to reach the extremes of temperature and pressure in order to test the theories and the equations used to describe the matter around him. In a fast expanding field a review and summary of research and research techniques is always desirable and it is for this reason that Brey's book is a welcome addition to any technical book shelf.

Given a system which is in equilibrium, reasoned Le Chatalier, and upon which a stress is impressed, the result would be a shift in the equilibrium in such a way as to relieve the stress. It is thus that at high pressures, molecules, forced together, polymerize, change state from gas to liquid or liquid to solid, and generally reach the points of discontinuity in properties associated with such changes. The scope of this book is centered upon materials in this critical condition. The author handles the presentation of the mass of work without becoming cumbersome and holds the readers interest.

In obtaining the pressures necessary to such research, equipment design is the limiting factor. The author discusses at some length the principles of pressure vessels and seals. Specific experimental equipment used by some high pressure researchers is mentioned but it is left to the reader to refer to the original work for full descriptions. It was interesting to note that the greater portion of the work described dealt with large pressure values, so much so that the consideration of negative pressures takes the reader somewhat by surprise. While little work is available in the latter area, this portion of the book could be expanded.

In all, the book is excellent for the engineer,

chemist, or physicist who requires an overall view of the pressure field or for the advanced student interested in pursuing the study of material behavior.

RICHARD HOLLINGER
The Franklin Institute Laboratories

THE METALLURGY OF VANADIUM, by William Rostoker. 185 pages, illustrations, 5¾ × 9 in. New York, John Wiley & Sons, Inc., 1958. Price, $8.50.

There are many metals whose names are familiar to us but we actually know very little concerning their physical or mechanical properties. Some of these, such as uranium, titanium, and zirconium, have become well known in the last few years because of their sudden increased utilization. This increased use has been accompanied by an intensive research effort resulting in many diverse publications. Thus, the appearance of books devoted to such metals has been the logical consequence of research, sharp increase in use, and widespread journal publications.

The present text does not result from such a pattern. Vanadium is still used in relatively small quantities and then only as an alloying element. The author points out that the metal itself or in the alloyed state has not as yet been used for industrial purposes. Furthermore, there has not been extensive research or publication concerning it.

The material for the book derives in large part from research at Armour Research Foundation sponsored by the Materials Laboratory of the Wright Air Development Center. Because of the limited information available on this metal, the book will probably require revision in a few years. There is, however, a sufficient amount of information to warrant publication.

In fact, the appearance of this book is encouraging for several reasons other than the presentation of the compilation of data. It shows that a government agency is willing to invest money for research on a metal for which there is not a guaranteed pay-off. It should be satisfying to the average metallurgist to be able to bring himself thoroughly up-to-date on a metal before its widespread application. Finally the book itself should stimulate thinking concerning the metal and thus result in additional research that is necessary to bring about its more extensive utilization.

The author covers the extractive metallurgy of vanadium, its properties, metallography, fabrication technology, corrosion behavior, alloying characteristics, and miscellaneous topics. He points out that many of the physical properties reported are tentative since they were made on vanadium more impure than is now available. However, the scope of physical properties covered is excellent. Properties that are not usually given in similar texts are presented; for instance, detailed structure, atomic, electrical and magnetic properties.

An informative chapter is devoted to the constitution of vanadium alloys. Although the data on the individual systems are meager, thirty-nine binarys and several ternarys are covered. Phase diagrams for some of them are shown.

The incomplete state of the technology of vanadium is well illustrated in the few pages devoted to its melting and casting. Since it has only been melted on a laboratory scale, the author discusses techniques which are used in the titanium industry. He quite aptly points out that these should be applicable to vanadium.

The references are given at the end of each chapter and appear to be up-to-date. There is a subject index. The book is concisely written, well illustrated, and should serve as a ready reference for the metallurgist.

R. L. SMITH
The Franklin Institute Laboratories

ONCE ROUND THE SUN, by Ronald Fraser. 160 pages, illustrations, 5½ × 8½ in. New York, The Macmillan Co., 1958. Price, $3.95.

Once Round the Sun is an innocuous title for a book on contemporary science, but despite the title this is a remarkable little book. Concentrated in its 160 pages is a veritable mine of information concerning scientific investigations during the 18-month IGY.

The author, Ronald Fraser, is Administrative Secretary of the International Council of Scientific Unions, the agency which sponsored the world-wide operation of the IGY. Thus in this position he is eminently qualified to assess and describe the various facets of sci-

ence under investigation. As part of his job he had to acquire a familiarity with every research project of every discipline that had any role in the IGY.

Essentially the book is a compendium in which thumbnail sketches of these researches are given. As an example, in Chapter I the author covers the size and shape of the earth, the spinning of the earth to give the day, the motion of the earth round the sun to yield seasons, the length of the day, the magnetic fields of the earth—now and in the past, precession, change in magnetic field, the wandering of the Poles, glaciers, and continental drift. Here are about a dozen subjects covered in a chapter of 15 pages! About a page per subject is the length of the discussion and herein lies the merit of this book. Over 125 topics are covered.

Wherein lies the value of a book of this type? This reviewer feels that this is one of the significant books to appear concerning the IGY. Only in this work by Ronald Fraser can the scientist find material on every line of research in the IGY. True, it is not comprehensive, but the author does not claim this for his book.

The author has performed a service to science in putting into print vignettes of the science panorama as touched in this period. It would be well if the author or someone in his position could publish a definitive picture of what man has tried to do in this IGY. Only when the full story is told will the average man—both layman and scientist—appreciate the extent of this international undertaking.

In time a work on this will be published. When it is it would be well to remember that Ronald Fraser pointed the way.

This reviewer considers *Once Round the Sun* one of the most significant books of the year.

I. M. LEVITT
Fels Planetarium of The Franklin Institute

ANNUAL REVIEW OF NUCLEAR SCIENCE, VOLUME 7, 1957, edited by J. G. Beckerley. 496 pages, diagrams, 6 × 8¾ in. Palo Alto (Calif.), Annual Reviews, Inc., 1957. Price: $7.00 (USA); $7.50 (foreign); postpaid.

With the appearance of the seventh volume in this series, one may assume that almost all potentially interested persons are now thoroughly familiar with the characteristics of these reviews. This volume is similar in content, format and quality to previous ones. It serves the same useful purposes, namely, it provides reviews useful to the expert and to the interested non-specialist. Copious literature references, tables and graphs make these reviews valuable reference articles.

It is to be noted that several of the chapters now show the impact of the discovery of the non-conservation of parity. Perhaps the most exciting chapter is that on hyperons and heavy mesons in which are discussed charge, parity and time invariance.

The titles of the articles are: Mu-Meson Physics, Radiochemical Separations by Ion Exchange, Equipment for High Level Radiochemical Processes, Cellular Radiobiology, Biochemical Effects of Ionizing Radiation, Vertebrate Radiobiology (Lethal Actions and Associated Effects), Vertebrate Radiobiology (The Pathology of Radiation Exposure), The Collective Model of Nuclei, Nuclear and Nucleon Scattering of High Energy Electrons, Collision of $\lesssim$ 1 BEV Particles (Excluding Electrons and Photons) with Nuclei, The Measurement of the Nuclear Spins and Static Moments of Radioactive Isotopes, Hyperons and Heavy Mesons (Systematics and Decay).

LEONARD MULDAWER
Temple University

THE PATH OF CARBON IN PHOTOSYNTHESIS, by J. A. Bassham and M. Calvin. 104 pages, illustrations, 5¾ × 8¼ in. Englewood Cliffs, N. J., Prentice-Hall, Inc., 1957. Price, $3.00.

This short monograph describes the excellent work done by Calvin, Bassham, and coworkers at the University of California, Berkeley, on the photosynthetic reaction, and particularly the carbon reduction cycle, the sequence of reactions leading from carbon dioxide to carbohydrates.

It contains an excellent short historical background on the problem, a description of the analytical procedures employed in the work and the plant materials, and discussion of the results. In addition to the work on the carbon reduction cycle, short discussions of other problems such as fat synthesis, the hydrogen transfer system, the quantum con-

version process and the oxygen evolution system are included. Areas of further work and gaps in our present knowledge are indicated.

In addition to being of great interest to workers in the field, the book is so well written as to serve the general scientific reader as well.

M. M. LABES
The Franklin Institute Laboratories

PRINCIPLES OF PHYSICAL CHEMISTRY, by Wallace S. Brey, Jr. 432 pages, 6 × 9¼ in. New York, Appleton-Century Crofts, Inc., 1958. Price, $7.00.

The technical worker has been forced more and more to use the principles of physical chemistry to study the molecular world. While it is neither practical nor necessary for every scientific worker to be a physical chemist, it has become desirable that he have a working knowledge of the applications of the field to his own. This book is designed for use as a text for a terminal course in physical chemistry as it applies to the biochemical field.

The presentation of thermodynamics is little different from that in most good texts and the development of equations in kinetics and equilibria is standard. As might be expected, colloid chemistry is considered in some detail, especially with regard to experimental techniques, and the field of radioactivity concentrates on tracer isotopes and their use. The chapters on chemical kinetics might have been more interesting for the biochemist if a discussion of the use of rate constants in determining the probable mechanism of enzyme catalysed reactions had been used.

The book achieves the author's aim for a lucid presentation of physical chemistry without undue mathematical work and should give the student a fair grasp of the subject. It should also be quite good for the physical chemist who has an interest in seeing how physical chemistry may be applied in another field.

RICHARD HOLLINGER
The Franklin Institute Laboratories

EMULSIONS, THEORY AND PRACTICE, by Paul Becher. 382 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1957. Price, $12.50.

The monographs of the American Chemical Society are intended to serve two principal purposes: "first to make available to chemists a thorough treatment of a selected area in form usable by persons working in more or less unrelated fields to the end that they may correlate their own work with a larger area of physical science discipline; secondly, to stimulate further research in the specific field treated."

Dr. Becher's book well meets these criteria. It is written to be read by chemists not versed in emulsion technology, and it includes extensive bibliographies for those who wish more details than can appropriately be included in the book.

Modern developments in the theory and practice of emulsions are clearly explained in this new monograph. It begins with the facts and theories of surface chemistry relevant to emulsions, and the physical properties of emulsions as a function of emulsion composition. Next, the theories of emulsion stability, and manifestations of instability (such as inversion, creaming, and complete demulsification) are covered. The volume also contains discussions of the chemistry of emulsifying agents, emulsifier efficiency as a function of composition, and the technique of emulsification, including the various types of emulsators. It offers practical formulation of numerous emulsion types, and describes the commercial aspects of demulsification.

Two useful appendices deal with the measurement of emulsion properties, and commercially available emulsifying agents.

Much of the material is based on developments of the last 15 years; earlier work, when significant, is cited. Here is coverage of the subject that will bring the cosmetic, food, petroleum, coating, and textile fields thoroughly up-to-date.

EDMUND THELEN
The Franklin Institute Laboratories

THE HYPERCIRCLE IN MATHEMATICAL PHYSICS, by J. L. Synge. 424 pages, diagrams, 6 × 9¼ in. New York, Cambridge University Press, 1957. Price, $13.50.

This book is the outgrowth of ten years' work at the Universities of Maryland, Brown, Harvard, MIT, Carnegie Institute of Technology, various European universities and the National Bureau of Standards.*

Let the reader recall that the equations of motion of an isotropic elastic solid are:

$$P_{1x} + S_{3y} + S_{2z} + \rho X = \rho U_{tt}$$
$$S_{3x} + P_{2y} + S_{1z} + \rho Y = \rho V_{tt}$$
$$S_{2x} + S_{1y} + P_{3z} + \rho Z = \rho W_{tt}.$$

The equations of equilibrium exist when $U_{tt} = V_{tt} = W_{tt} = 0$. Here the body force per unit mass at (x, y, z) has components X, Y, Z and ρ is the density of the body. P_1, P_2, P_3 are tensional stresses, S_1, S_2, S_3 are shear stresses, u, v, w are displacements in space

$$P_{1x} = \frac{\partial P_1}{\partial x}, \quad P_{2y} = \frac{\partial P_2}{\partial y}, \quad P_{3z} = \frac{\partial P_3}{\partial z}, \quad S_{1y} = \frac{\partial S_1}{\partial y}, \quad S_{2z} = \frac{\partial S_2}{\partial z} = S_{3x} = \frac{\partial S_3}{\partial x}$$

$$S_{1z} = \frac{\partial S_1}{\partial z}, \quad S_{2x} = \frac{\partial S_2}{\partial x}, \quad S_{3y} = \frac{\partial S_3}{\partial y}$$

$$U_{tt} = \frac{\partial^2 u}{\partial t^2}, \quad V_{tt} = \frac{\partial^2 v}{\partial t^2}, \quad W_{tt} = \frac{\partial^2 w}{\partial t^2}.$$

If S = strain energy per unit volume then

$$2S = P_1\alpha + P_2\beta + P_3\gamma + S_1a + S_2b + S_3C.$$

Let a particle originally at (x, y, z) be displaced to $(x + u, y + v, z + w)$. Then the three extensioned strains are

$$\alpha = \frac{\partial u}{\partial x}, \quad \beta = \frac{\partial v}{\partial y}, \quad \gamma = \frac{\partial w}{\partial z}$$

and the three component shear strains are

$$a = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \quad b = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}, \quad c = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}.$$

The relations between stresses and strains are:

$E\alpha = P_1 - \sigma(P_2 + P_3)$	$na = S_1$	E = Young's modulus
$E\beta = P_2 - \sigma(P_3 + P_1)$	$nb = S_2$	σ = Poisson's ratio
$E\gamma = P_3 - \sigma(P_1 + P_2)$	$nc = S_3$	n = modulus of rigidity
$\Delta = \alpha + \beta + \gamma$; $P_1 + P_2 + P_3 = 3k\Delta$		Δ = dilatation
$3k = E(1 - 2\sigma)^{-1}$; $2n = E(1 + \sigma)^{-1}$		$-\Delta$ = cubical compressions
		k = bulk modulus

In the Prager-Synge concept, to each state of stress of the body (six functions of three coordinates) there corresponds a single point of function-space. This space is endowed with a metric in a way which seems very natural physically, the square of the distance of a point from the origin of function-space (the state of zero stress) being twice the strain energy of the state corresponding to that point.

This representation of states of stress by points of function space gives rise to the notion of hypercircle. The hypercircle of function-space appears in the theory as the basis of possible positions of the point corresponding to the unknown solution of a problem of elastic equilibrium.

The analytic problem can be presented as the geometrical problem of finding the point of intersection of two orthogonal linear subspaces in a suitably chosen function-space.

* The first papers on the subject were: PRAGER, W. AND SYNGE, J. L., "Approximations in Elasticity Based on the Concept of Function Space," *Quart. Appl. Math.*, Vol. 5, pp., 241–69 (1947); and SYNGE, J. L., "The Method of the Hypercircle in Function-Space for Boundary Value Problems," *Proc. Roy. Soc. A.*, Vol. 191, pp. 447–67 (1947).

In Chapter 1, "Geometry of Function Space Without a Metric," Professor Synge presents the notions of P-space (physical space in which the problem is set up) and function-space vectors. If P-space is of n dimensions specified by $x^1, x^2, \cdots, x^n$ co-ordinates and we consider a set of m functions of these co-ordinates—$S_1, S_2, \cdots, S_m$—then a function-space vecter $\vec{S}(S_1, S_2, \cdots, S_m)$ is said to correspond to this set of functions.

To illustrate these ideas, in treating an elastic body we would have $n = 3$ (dimensionality of ordinary space) and $m = 6$ (number of stress components). The zero F-vector $\vec{0}$ corresponds to the absence of stress and two vectors are equal if, and only if, they correspond to the same state of stress throughout the body.

No knowledge of function-space on the part of the reader is assumed and he is carefully carried through the concept of linear subspaces and their intersections.

In Chapter 2, "Geometry of Function-Space with Positive Definite Metric," Professor Synge emphasizes that for his purposes

(*i*) P-space is a portion of Euclidean Space (of any number of dimensions), bounded by surfaces composed of a finite number of portions, each with continuous normals.

(*ii*) Admissible functions are piecewise continuous, except possibly for a finite number of singular points, and the squares of the functions are integrable. At discontinuities a function has an unassigned value and two functions are regarded as identical if their values agree at all points of continuity.

In this chapter he rigorously establishes the concepts of orthogonal linear subspaces, hyperplane, hypersphere and hypercircles. These having been established, he considers the problem: "In function-space with positive-definite metric, there are two intersecting orthogonal linear subspaces L', L''. We want to find their point of intersection S."

In the method of the hypercircle only a finite number of points are used, some in one of the two linear subspaces and some in the other. In general, a limiting process is not used and the actual solution (i.e., the point of intersection of the two linear subspaces) is not found. Although it is not found, something is learned about its position, namely, that it is located on a certain hypercircle in function-space. This provides inequalities bounding the solution in a mean square sense (and in some cases pointwise) and if the radius of the circle is sufficiently small, the centre of the circle, or any point on it, is a good approximation to the unknown solution.

The remaining chapters on the positive definite metric are concerned with "Dirichlet-Neumann problems."

The general title "Dirichlet-Neumann problems" may be used to cover the following problems: P-space is a portion of Euclidean N-space bounded by a surface B, and we seek solutions of the Laplace equation

$$\Delta u = 0$$

satisfying one or other of the following boundary conditions:

Dirichlet problem:

$$(u)_B = f,$$

Neumann problem:

$$(\partial u/\partial n)_B = g, \quad (\int g dB = 0),$$

First mixed problem:

$$(u)_{B_1} = f, \quad (\partial u/\partial n)_{B_2} = g, \quad B = B_1 + B_2,$$

Second mixed problem:

$$\left(\alpha u = \beta \frac{\partial u}{\partial n}\right)_B = f.$$

Here Δ is the Laplace operator $\Delta = \dfrac{\partial^2}{\partial x_i \partial x_i}$, where x_i are rectangular Cartesian coordinates and the suffixes take the values 1, 2 with summation understood for repeated suffixes.

f, g, α, β are given on B. It is clear that the second mixed problem includes the first as a particular case (put $\alpha = 1, \beta = 0$ on B_1, $\alpha = 0, \beta = 1$ on B_2), and that the first mixed problem contains the Dirichlet and Neumann problems as particular cases.

In Chapter 3 "The Dirichlet problem for a finite domain in the Euclidean Plane," Synge devotes his attention to applying the hypercircle method (using a considerable amount of arithmetical detail) to the Dirichlet problem in the plane.

He first establishes in notation suitable for the hypercircle method the classic relation "The integral of the square of the gradient of the solution of the Dirichlet problem is less than the integral of the square of the gradient of any other function which satisfies the boundary condition" (compare O. D. Kellogg, "Foundations of Potential Theory," p. 279).

By locating the solution of a Dirichlet problem on a hypercircle, Synge succeeds in bounding the gradient of the solution in a mean-square sense. He then proceeds to bound the solution and its derivatives at a point. This pointwise bounding starts only after the solution has been located on a hypercircle. He illustrates the details with the example of a deformed tubular condenser.

He next introduces the concepts of pyramid functions and pyramid F-vectors. Consider a plane polygon with a point A inside it. Now raise the point A out of the plane of the polygon, the displacement (of length h) to the new position A' being perpendicular to the plane of the polygon. Join A' to the corners and sides of the polygon. This creates a pyramid, having A' for vertex and the polygon for base.

Let x_i be rectangular Cartesian coordinates in the plane of the polygon, and let $x_i = a_i$ at A. Let $t(x_1, x_2)$ be the height of the point on the surface of the pyramid directly above the point x_i in the plane. It is an easy function to think about geometrically, for the pyramid is a three-dimensional graph of the function $t(x_1, x_2)$. We write $t = 0$ for points outside the polygon.

The following facts are obvious:

(*i*) $t(x_1, x_2)$ is a continuous function in the whole plane.

(*ii*) $t = 0$ on and outside the polygonal boundary.

(*iii*) $t = h$ when $x_i = a_i$.

(*iv*) The partial derivatives $t_{,i}$ are zero outside the polygon and have constant values in each of the triangles formed by joining A to the corners of the polygon.

(*v*) t is of the form

$$t(x_1, x_2) = h\tau(x_1, x_2),$$

where τ is independent of h, depending only on the geometry of the polygon and the position of A inside it.

We call $t(x_1, x_2)$ a pyramid function. t is a function of position in the plane of the polygon and $t_{,i}$ a vecter field in that plane.

Suppose that we are dealing with a Dirichlet problem for P-space V, a region bounded by a curve B; V may be simply or multiply connected. Let us draw a polygon lying entirely in $V + B$ and generate in it a pyramid function $t(x_1, x_2)$. Then $t_{,i}$ corresponds to a homogeneous associated F-vector T'. Such an F-vector is called a pyramid F-vector of the first class. Other F-vectors are defined, namely F-vectors of the second class, strip F-vectors, hexagonal pyramid F-vectors of the first and second classes and square pyramid F-vectors.

The important fact which makes pyramid F-vectors useful is the following: Given any function u, continuous and with continuous partial derivatives in a region V bounded by a curve B, we can approximate as closely as we like to both u and its partial derivatives $u_{,i}$ by a linear combination of pyramid functions.

Synge applies these concepts with considerable attention to numerical details to the examples of an hexagonal condenser and a square condenser.

Chapter 4 is "The Torsion Problem." The problem of the twisting or torsion of an elastic cylinder is a classic problem in the theory of elasticity (compare A. E. H. Love "Mathematical Theory of Elasticity" p. 310). For a simply connected cross-section it is mathematically identical with other physical problems—fluid in a rotating container, viscous flow through a pipe and the equilibrium of an elastic membrane.

The torsion problem for a multiply connected section reduces to finding the intersection of two orthogonal linear subspaces, L', L''. When the section is simply connected, the torsion problem is a Dirichlet problem and the pyramid F-vectors are consequently available for use. Synge next shows that the pyramid F-vectors may be used in the case of a multiply-connected section.

The torsion problem for a beam of regular hexagonal cross-section lends itself to treatment by hexagonal pyramid F-vectors. Synge makes a most extended and detailed analysis of this problem. He concludes Chapter 4 with a similar detailed analysis for the torsion of a hollow square.

Chapter 5 deals with various boundary value problems. The systematic approximation to the solution of a plane Dirichlet problem was based on pyramid F-vectors. In Chapter 5 Synge shows how this idea is carried into 3-space for the Dirichlet and Neumann problems. He makes extended and detailed analyses of the problems of a viscous flow in a channel, of a membrane supported elastically, of a cube deformed by its own weight, of a beam bent by terminal forces, of viscous flow across a tank and a plate under pressure.

Chapters 6 and 7 deal with the geometry of function-space with indefinite metric. These two chapters are general in their discussion and lack the pungent flavor of the meaty examples given in the chapters on geometry of function space with definite metric. Some of this later material on elastic and electromagnetic vibrations appears in the Proceedings of the Eighth Symposium on Applied Mathematics of the American Mathematical Society (Chicago, 12 April 1956).

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PUBLICATIONS RECEIVED

The following books have been received for review in the JOURNAL. Lack of space prevents more than this listing.

DESIGN OF CONCRETE STRUCTURES, by Leonard Church Urquhart, Charles Edward O'Rourke and George Winter. Sixth edition, 546 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $8.00.

SYSTEMATIC SOCIOLOGY, by Karl Mannheim, edited by J. S. Eros & W. A. C. Stewart. 169 pages, 5¼ × 8¼ in. New York, Philosophical Library, Inc., 1958. Price, $6.00.

PHYSICS, by Henry Semat and Robert Katz. 927 pages, diagrams, 6 × 9 in. New York, Rinehart & Company, Inc., 1958. Price, $9.00.

THE ENCYCLOPAEDIA OF RADIO AND TELEVISION, Technical Consultant, J. H. REYNER. 736 pages, diagrams, 5¼ × 8¼ in. New York, Philosophical Library, Inc., 1958. Price, $12.00.

FUNDAMENTAL CONCEPTS OF INORGANIC CHEMISTRY, by Esmarch S. Gilreath. 421 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $7.50.

THE THEORY OF FUNCTIONS OF A REAL VARIABLE AND THE THEORY OF FOURIER'S SERIES, VOLUME I, by E. W. Hobson. Unabridged and unaltered republication of the first edition, 736 pages, 5½ × 8½ in. New York, Dover Publications, Inc., 1957. Price, $3.00 (paper).

THE THEORY OF FUNCTIONS OF A REAL VARIABLE AND THE THEORY OF FOURIER'S SERIES, VOLUME II, by E. W. Hobson. Unabridged and unaltered republication of the first edition, 780 pages, 5½ × 8½ in. New York, Dover Publications, Inc., 1957. Price, $3.00 (paper).

FOUNDATIONS OF PHYSICS, by Robert Bruce Lindsay and Henry Margenau. Unabridged republication of the first edition, 542 pages, 5½ × 8½ in. New York, Dover Publications, Inc., 1957. Price, $2.45 (paper).

ELEMENTS OF MATHEMATICAL BIOLOGY, by Alfred J. Lotka. Unabridged republication of the first edition, 465 pages, diagrams, 5½ × 8 in. New York, Dover Publications, Inc., 1956. Price, $2.45 (paper).

THERMODYNAMICS, by Enrico Fermi. Unabridged and unaltered republication of the first edition, 160 pages, 5½ × 8 in. New York, Dover Publications, Inc., 1956. Price, $1.75 (paper).

MATHEMATICAL FOUNDATIONS OF INFORMATION THEORY. A translation of two papers by A. I. Khinchin. 120 pages, 5½ × 8 in. New York, Dover Publications, Inc., 1957. Price, $1.35 (paper).

GASEOUS CONDUCTORS—THEORY AND ENGINEERING APPLICATIONS, by James Dillon Cobine. Unabridged republication of the first edition with corrections by the author, 606 pages, 5½ × 8 in. New York, Dover Publications, Inc., 1958. Price, $2.75 (paper).

TEACHING SCIENCE TO THE ORDINARY PUPIL, by K. Laybourn and C. H. Bailey. 415 pages, diagrams, 5½ × 8½ in. New York, Philosophical Library, 1957. Price, $10.00.

ELECTRONIC HOBBYISTS' HANDBOOK, by Rufus P. Turner. 160 pages, diagrams, 5½ × 8½ in. New York, Gernsback Library, Inc., 1958. Price, $2.50 (paper).

ENCYCLOPEDIA OF CHEMICAL REACTIONS, compiled by C. A. Jacobson, edited by Clifford A. Hampel. 479 pages, 6 × 9 in. New York, Reinhold Publishing Corp., 1958. Price, $12.75.

ANALYTICAL MECHANICS FOR ENGINEERS, by Fred B. Seely, Newton E. Ensign and Paul G. Jones. Fifth edition, 475 pages, diagrams, 5½ × 8½ in. New York, John Wiley & Sons, Inc., 1958. Price, $7.25.

OUR NUCLEAR ADVENTURE, ITS POSSIBILITIES AND PERILS, by D. G. Arnott. 170 pages, diagrams, 5½ × 8½ in. New York, Philosophical Library, 1958. Price, $6.00.

HANDBOOK OF SUPERSONIC AERODYNAMICS, compiled and edited by the Bureau of Ordnance. v.p., 8 × 10½ in. Washington, D. C., Government Printing Office, 1957. Price, $1.50.

THE MEASUREMENT OF COLOUR, by W. D. Wright. 263 pages, diagrams, 5½ × 8½ in. New York, The Macmillan Company, 1958. Price, $10.75.

ORAL COMMUNICATION OF TECHNICAL INFORMATION, by Robert S. Casey. 199 pages, diagrams, 5 × 7½ in. New York, Reinhold Publishing Corp., 1958. Price, $4.50.

SPRAYING TECHNIQUES IN AGRICULTURE, Monograph 2, a symposium. 117 pages, diagrams, illustrations, 5½ × 8½ in. New York, The Macmillan Company, 1958. Price, $4.00.

THERMODYNAMICS OF ENGINEERING SCIENCE, by S. L. Soo. 620 pages, diagrams, 6 × 9 in. New York, Prentice-Hall, Inc., 1958. Price, $9.50.

ESSENTIAL BUSINESS MATHEMATICS, by Llewellyn R. Snyder. Third edition, 470 pages, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1958. Price, $5.50.

BASIC PRINCIPLES OF CHEMISTRY, by J. Rae Schenck and Raymond M. Martin. 400 pages, 6 × 9 in. New York, Prentice-Hall, Inc., 1958. Price, $6.75.

PETROLEUM REFINERY ENGINEERING, by W. L. Nelson. Fourth edition, 960 pages, diagrams, illustrations, 6 × 9 in. McGraw-Hill Book Co., Inc., 1958. Price, $15.00.

NOTE: The review of "The General Theory of Electrical Machines," by Bernard Adkins, which appeared on p. 355 of our April 1958 issue, listed "Price not given." The publisher, John Wiley & Sons, Inc., has informed us that the price is $8.00.

CURRENT TOPICS

Shift Register Transistor.—A "significant breakthrough" in research that may lead to major reduction in the size and complexity of computers, remote control equipment, and other large electronic systems was announced recently by Dr. Irving Wolff, Vice President, Research, of RCA Laboratories.

The advance has been achieved in research aimed at developing a new class of ultra-miniature "integrated" electronic devices that will combine many different functions in a few small components, according to Dr. Wolff.

"A long forward step has now been taken with an experimental shift register transistor under development by Dr. J. T. Wallmark, of the RCA Laboratories technical staff at the David Sarnoff Research Center," he said.

"This laboratory device, only half an inch long and only four-thousandths of an inch thick, is expected with further development to perform in one application functions that now require a circuit arrangement of twenty transistors, forty resistors, and twenty capacitors. The replacement of transistors by a single ultraminiature device is striking enough, but an even greater achievement is the replacement of capacitors and resistors by this integrated device. In this respect, it embodies an entirely new concept in electronic technology."

Dr. Wolff stated that laboratory operation of the shift register transistor has now shown the feasibility of integrating not only the tube and transistor components which are the active elements in a circuit, but the capacitors and resistors—the passive elements—as well.

"With the principle successfully established, the way is open to the development not only of an integrated shift register transistor, but of an entire class of integrated devices incorporating new standards of miniaturization, economy, and reliability," he said.

"Until now, our efforts to cope with the growing size and complexity of computers and other major electronic systems have been concentrated on miniaturizing and simplifying standard components. At the same time, however, the total number of components has steadily increased, raising new problems of complexity and reliability. This principle of integrating many functions into a single small component should enable us to reverse this trend and to progress toward the development of more economical, versatile, and reliable systems of all types."

Dr. Wallmark described the function of a shift register as the short-term storage of a multiple-digit number or similar information, for subsequent transfer, intact, at the right moment in the computing process. In this way, the information can be related to other data originating elsewhere in the system at a different time, or delayed for delivery at the output as it is needed. In present systems, according to Dr. Wallmark, this function is performed by complex shift register circuits employing up to 20 electron tubes or transistors with their accompanying capacitors and resistors.

Dr. Wallmark gave this description

of one way in which a shift register transistor may be used:

The ten transistor-like elements on the single small strip of germanium function as ten separate two-way switches connected in series. Each of these elements can receive and hold a single bit of information in the form of a strong or a weak current, corresponding in computer language to "0" or "1". The series of ten permits storage of a ten-digit number made up of various combinations of these two symbols.

The bits of information are fed into the device one by one at high speed at one end, and are shifted from one element to the next, in order, by electrical "shift pulses" passed through the germanium. When the "shift pulses" are stopped, each bit of information will remain in one of the transistor-like elements. When the pulses are re-started, each bit shifts from one element to the next until the pulses are again stopped. When the bits reach the final element, they are "read out" in the same order in which they were fed in at the start. In this way, the information is kept intact, and yet its passage is delayed as long as necessary.

"Such a device represents integration not only of ten different positions now requiring two transistors each, but also of the circuits associated with these different positions," said Dr. Wallmark.

"Auto-Abstract."—An electronic machine that will read a magazine article and then write an abstract of it was disclosed at the national IRE convention recently. The system, called "Auto-Abstract," will be particularly helpful in scanning and excerpting scientific papers and foreign language articles.

H. P. Luhn of the IBM Corporation told an audience of engineers at the Convention that the complete text of an article is first transcribed onto magnetic or punched tape in a code that can be understood by an electronic data processing machine, such as the IBM 704 computer. The machine then analyzes the text word by word to derive statistical information concerning the frequency and distribution of the words in the text.

From this, the machine determines the relative degree of significance of the words and then grades each sentence as to its importance. Sentences scoring highest in significance are automatically extracted from the text and printed out by the machine to form the "Auto-Abstract."

In addition to creating abstracts, the system can be used to condense lengthy reports.

Abstracting is an important activity in scientific circles, requiring the services of technically skilled people. The Auto-Abstract system offers the possibility of relieving these people of the chore so that they may devote more of their time to scientific work.

The system also promises to expedite the translation of foreign scientific articles by producing foreign language abstracts of the original paper. A person then only has to translate the abstract rather than the full paper to find out what is in it, and the translation of full articles can be limited just to those that are found to be especially important.

Control of Fusion through Electronics.—Controlled thermonuclear fusion, in which electronic techniques play a key role, may some day provide electric power from plants producing one million or more kilowatts from only half a pound of fuel a day, a scientist of the Radio Corporation of America predicted in a talk before

IRE annual convention. E. W. Herold, Director of the Electronic Research Laboratory, RCA Laboratories, said that the major obstacles to fusion control may be overcome in the laboratory within five to ten years, leading to "practical utilization" after several more decades. Then, with power so cheap and plentiful, he added, "we may speculate upon completely new methods of transmission."

"Since the cost, size and weight of the fuel are all negligible, the economic factors favor very large installations, perhaps exceeding the largest now known for electric power," said Mr. Herold. "The major cost of useful power will be the transmission system; much will have to be done by the electrical engineering profession to improve wire-line transmission.

"In addition, the era of fusion power may make this power so cheap and plentiful that we can afford to explore completely new transmission methods, perhaps using induction fields over short distances, or electro-magnetic radiation over longer distances. This day, should it come, may see our earth less dependent on the sun, whose inefficient but powerful radiant transmission system has been serving us so well for the past billion years."

Discussing the fusion process, Mr. Herold explained that the major objective in the production of power is to create extremely high temperatures in which nuclei of heavy hydrogen are forced together to form helium-4, releasing large amounts of energy. He pointed out that the particles must then be held together long enough and in a fixed volume so that they can react to produce a steady flow of energy. Methods under investigation, according to Mr. Herold, involve confining heavy hydrogen gas in a strong magnetic field and raising the gas to temperatures of 100,000,000 degrees or more by application of high-power induction heating.

"Evidently, a fusion reactor is what physicists have called an electrodeless gas discharge, and the ignition process is most conveniently performed by electric induction or some such means," he said. "It is these factors which make fusion power of such direct interest to the radio and electronics engineer."

Mr. Herold identified several fields in which the radio engineer and the electronics industry can contribute to the solution of the major problems for the future in fusion power research. Among these, he listed the desirability of "a major invention to reduce the power for a steady magnetic field over a volume of many cubic feet," and an ignition technique that will "fire up" the gas in a short time. He pointed also to a need for radiation-resistant materials, and "ideas on the direct conversion of fusion power to electricity."

"Controlled thermonuclear fusion in gas discharges offers hope of a revolutionary new energy source," he concluded. "However, major containment and heating problems are an obstacle, and achieved results are factors of a hundred below objectives. It is probable that these obstacles will be overcome, in the laboratory, within 5 to 10 years, but practical utilization may require a period of several decades beyond this. The radio and electronics engineer has an opportunity to make major contributions, possibly to shorten the time appreciably."

New High Temperature Alloy.—A new, improved high temperature alloy known as Carpenter Lapelloy "C" has been developed by The Carpenter Steel Company, Reading, Pa., for jet engine parts requiring excellent ductility and good impact resistance.

Carpenter Lapelloy "C," U. S. Patent No. 2,816,830, has a more homogeneous structure and more uniform fabrication behavior and mechanical properties than the conventional Lapelloy. It is used for highly stressed parts involving service temperatures up to 1200° F. Lapelloy "C" has greater ductility and impact strength at both room and elevated temperatures than conventional Lapelloy when heat treated to the same strength level.

High mechanical properties can be developed by heat treatment. It has good resistance to scaling and oxidation for continuous service up to 1400° F. The alloy offers best corrosion resistance when in the hardened and tempered condition. It is used for compressor wheels, turbine shafts, compressor buckets, blades, and bolts.

Type analysis of Carpenter Lapelloy "C" is: Carbon, 0.20–0.25%; Manganese, 0.65–1.00%; Silicon, 0.50% max.; Chromium, 11.00–12.00%; Nickel, 0.50% max.; Molybdenum, 2.50–3.00%; Copper, 1.75–2.25%, and Nitrogen, 0.06–0.10%.

Lapelloy "C" is made by Carpenter's Mel-Trol process, a new quality control system with patented ingot design. Mel-Trol is used to minimize the causes of inconsistent centerline quality which so often causes erratic service in alloys of this type made by conventional methods.

Lapelloy "C" is readily machinable; in the annealed condition it is comparable to Type 420 stainless steel. It is available in billets, hot-rolled and cold-finished bars, wire, cold-rolled strip, and special shapes.

Chemical Compound Aids Resistance to Radiation Effects.—A compound which is believed capable of doubling man's resistance to the effects of radiation has been developed at government laboratories in Oak Ridge, Tenn. The information was revealed by Dr. K. C. Atwood III of Oak Ridge at a news conference following presentation of a paper, "The Cellular Lesion in Radiation Injury," at the Symposium on Genetics in Medical Research at the University of Wisconsin.

Dr. Atwood indicated that the compounds AET, technically called aminoethylthiouronium, and related substances, have been developed to the point where it is now ready for human use should the need arise, and can be produced in quantity. It must, however, be taken prior to radiation exposure.

Dr. Atwood cautioned, "In my opinion, it will never be possible to completely obviate the effects of radiation. It may be possible to develop resistance to radiation dosage up to perhaps five times the normal tolerance of a human, but that's all.

"A strong enough dose would produce such a reaction in the human body that a complete rebuilding of man's molecule system would be necessary. This is of course impossible."

Tracking System for Moon Radar.—A punch card operated steering system for a moon radar antenna that is twenty-five times more accurate than previous electronic systems was revealed at the recent IRE National Convention.

The system, described by Olaf A. Guzmann, of the U. S. Army Signal Corps Engineering Laboratories, Fort Monmouth, N. J., can be applied in modified form for tracking other celestial bodies, including satellites.

Naval Observatory data regarding the orbit of the moon is punched into cards and fed to a digital computer which then calculates where the moon will be at different instants with respect to the antenna. This informa-

tion is stored on magnetic tapes and fed into an analog conversion system which provides a continuous flow of positional signals that keep the antenna aimed constantly and accurately at the moon as it moves across the sky.

Techniques for bouncing radar signals off the moon, pioneered by the Signal Corps in 1946, have recently taken on added importance in radio communications and radio astronomy. Last year the Naval Research Laboratory succeeded in receiving voice signals that were reflected by the moon, opening up the possibility of using the moon to relay long-distance radio transmissions. More recently the same group used moon radar to measure the distance to the moon with an accuracy of within 1000 ft.

New Passive Semiconductor Component.—A two-terminal, passive semiconductor component having novel and highly useful characteristics and known as a field effect varistor, has been developed by R. M. Warner, Jr., H. A. Stone and E. I. Doucette of Bell Telephone Laboratories.

This component has a constant-current feature which makes it ideally suited for a current regulator in circuits where either the load or supply voltage vary over wide limits. It can also be used as a current limiter or pulse shaper. Its a-c. impedance is very high, making it useful as a coupling choke or an a-c. switch.

The device, closely related in principle to the field effect transistor, contains a single planar junction which is made by diffusion. Current passes parallel to this junction through a constricted region called the channel. As the voltage across the device is increased, current increases and a depletion layer builds up which eventually reaches through the entire thickness of the channel. At this point, called the "pinch-off" point, a further increase in voltage does not produce any increase in current. Eventually an avalanche breakdown occurs, as the voltage is increased still further. Between the pinch-off and breakdown points, the current is essentially constant, and this is the region of maximum interest.

At present the varistors are fabricated by cutting dice from a slice of germanium or silicon containing a single diffused junction. The dice are heavily plated on all surfaces, and a circular trench is then cut into the diffused layer to within about 0.1 mil of the junction. This cutting requires a high degree of precision, since the characteristics of the final device depend heavily on the spacing between the bottom of the trench and the junction. In exploratory work, the trench is first cut with an ultrasonic tool and finished to the desired depth by etching. Leads are then attached by thermo-compression bonding or other convenient means.

Characteristics of the device can be altered by varying such parameters as channel depth, impurity gradient, length and width of channel, and selection of semiconductor material. Using silicon, units have been fabricated with a regulated current of one milliampere, pinch-off voltage of 10 volts and breakdown of 150 volts.

Current can be held constant to within 1% over a voltage range of 20 to 120 volts. Germanium units with a rating of 10 ma., pinch-off of 10 volts and breakdown of 25 volts have been made. It appears feasible at present to produce varistors which regulate current at any level between 10 microamperes and 10 milliamperes, and improvements in fabrication techniques should make higher current levels feasible.

For circuit applications, a parasitic

shunt capacitance of the order of a few mmfd. is present, which must be taken into consideration. In the constant current region, the ratio of a-c. impedance to d-c. resistance is typically 100 and may run as high as 1000, making it ideal as a coupling device. It differs from the conventional choke in that its a-c. impedance is constant over an appreciable frequency range.

Shock Test Machine for Missile Components.—A shock machine specifically designed for testing ballistic missile components, yet versatile enough for any type of shock testing, has been announced by Lycoming Division, Avco Manufacturing Corporation.

The machine was developed by the Research and Advanced Development Division of Avco, prime contractor for development of the re-entry vehicle for the Air Force Titan ICBM, and will be manufactured and marketed by Lycoming Division. It is the nation's first shock machine specifically developed to meet the rigid specifications of the WS-107A-2 Environmental Test Requirements—Airborne Electronic Equipment. The machine has proved a precision instrument during extensive environmental laboratory testing.

The Avco machine can produce, and accurately reproduce, sawtooth-wave shock patterns of more than 100 g's over a shock response spectrum of 80 to more than 1000 cycles per second. It provides a specified terminal-peak sawtooth pulse shape rising to 100 g's in 6 milliseconds and then dropping abruptly to zero. It can also provide a variety of wave patterns including quarter- and half-sine wave shocks.

From a pre-determined drop height, the carriage impacts against a molded lead pellet. By controlling the size and shape of these pellets, a wide variety of pulse forms can be obtained. The pellets themselves are uniformly produced by a simple molding process.

Precision operation is accomplished by pneumatically positioning the specimen carriage and releasing it for fall by a solenoid-operated release mechanism. This pellet is placed on the impact anvil set in the 2000 pound concrete base of the machine. Operator safety is insured by the built-in safety features of the simplified control panel. The anvil, carriage and superstructure are seismically isolated from each other to prevent ringing.

Avco's machine accommodates specimen weights up to 40 lb. and a cross-sectional area up to 12½ in. square. It is an ideal device for testing practically every item of electronic and mechanical equipment used in the most advanced ballistic missiles and other equipment subject to high impact loads.

Announcing the machine, Avco prototype development engineers stressed that the shock machine for 40-lb. specimens is a rugged device capable of generating and duplicating a wide variety of shock waves. Installation is simple with no special foundations required; operations and maintenance are economical and the machine requires a minimum of floor area.

Device Measures Low Radioisotope Concentrations.—An instrument so sensitive it can detect a thousandth of a gram of chromium-51 dissolved in the Columbia River has been developed by General Electric Company scientists at the Hanford atomic plant.

Developed to measure extremely small amounts of radioactive elements in the Columbia, the device is capable of measuring lower radioisotope concentrations than those occurring naturally in common foods such as meat, milk and vegetables.

The electronic wizard identifies and provides information for calculating the concentrations of 11 different isotopes in a sample by means of an energy analysis of the sample's gamma rays.

To do this the instrument uses gamma ray and coincidence gamma-ray spectrometry—relatively new tools of atomic scientists for detecting radiation and for identifying and measuring each contributing radioisotope.

In a few ounces of water the instrument has identified and measured by means of their radiations, minute quantities of sodium-24, manganese-56, copper-64, arsenic-76, chromium-51, neptunium-239, zinc-65, scandium-46, barium-140, cobalt-60 and manganese-54.

Before the development of this new Hanford instrument, such quantitative measurement was impossible without chemical separation, a tedious, involved process.

The instrument, developed in General Electric's Hanford laboratories, consists of a probe containing a sample of river water, boiled until only its dissolved solids remain.

This probe is placed inside a heavily shielded safe-like cell which reduces the interference from the "natural background" radiation.

Information gained by the probe—an activated sodium iodide crystal and a multiplier phototube sealed in an aluminum can—is relayed to a rack of instruments where this information is analyzed.

Mr. R. W. Perkins, a General Electric chemist at the Hanford plant, performed the analytical development work on the new instrument.

DO YOU KNOW OF THE FRANKLIN INSTITUTE'S NEED FOR GIFTS?

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